A HEURISTIC ALGORITHM FOR SOLVING LARGE-SCALE BI-OBJECTIVE VEHICLE ROUTING PROBLEMS WITH TIME WINDOWS

Konstantinos G. Zografos
Professor

Kostantinos N. Androutsopoulos
Research Associate

Athens University of Economics and Business
Transportation Systems and Logistics Laboratory
76 Patision Str.
104 34 Athens-Greece
Tel.: (30 1)8203167-433, Fax: (30-1)8203444
e-mail: kostas.zografos@aueb.gr

Extended Abstract

The performance of the logistical operations of a given company determine to a great extent the level of service offered by the company to its customers. One of the most important and crucial logistical functions is the transportation and distribution of goods to the customers.

The objective of a distribution system is to guarantee that a product is available at the right place, the right time, at right conditions, at the right price. A type of decision of major importance that directly affects the performance of a distribution system is the routing and scheduling of delivery trucks. It has been widely stated in the literature that this type of decision is modelled by difficult to solve combinatorial optimisation problem [Solomon et al, 1987]. As a consequence exact solution methods cannot find the optimum solution even for moderate size problems. Alternatively several heuristic approaches have been developed within the last two decades aiming at identifying near optimal solutions for both problem categories. The objective of this paper is to formulate the bi-objective vehicle routing and scheduling problem as integer programming model and present a new heuristic algorithm for solving it.

In general the Vehicle Routing and Scheduling problem for the distribution of goods could be defined as the determination of the routes assigned to a fleet of vehicles for the transportation of products from a number of depots to a set of demand points (customers). More specifically the Vehicle Routing and Scheduling problem can be decomposed into the following two subproblems: (i) the assignment of a set of demand points to each vehicle and (ii) the identification of the order in which each vehicle should visit the demand points i.e. routes [Laporte, 1992]. The basic characteristics of Vehicle Routing and Scheduling problem are: a) the vehicle capacity constraint which implies that the total demand covered by each vehicle should not exceed the vehicle's capacity, b) the route duration constraint which imposes a time limitation on the use of the vehicles and consequently on the duration of the routes, and c) the time windows constraint which refers to the limitation that is imposed on the earliest and latest delivery time at each point of demand.

Based on the criteria used and constraints imposed for the identification of the routes of the vehicles, several categories of the Vehicle Routing and Scheduling problems emerge. A classification of the Vehicle Routing Problem can be derived by considering the following characteristics: i) constraints related to the depot(s) and/or the points of demand e.g. the number of depots, time windows etc, ii) the features of the fleet, e.g. the number of vehicles comprising the fleet, the route duration, etc., iii) characteristics that have no relation with either the depots and points of demand or the fleet e.g. the service policy of the company, iv) the criteria for selecting the routes e.g. cost and/or risk [Desrochers, 1990].

The focus of this paper is on the bi-obejctive Vehicle Routing problem with time windows which can be defined as follows: "Given a set of M depots, N-M points of demand d_j each, a fleet of NV vehicles with capacity K_v each and $\{[a_i,b_i], a_i>0 \text{ and } b_i>0\}$ a set of time windows identify the routes of the vehicles that minimize the total travel cost of the vehicles. The mathematical model that expresses the Vehicle Routing Problem with Time Windows is defined by the constraints (1-7) [Solomon,1988].

$$Minimise\{w_1 \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{v=1}^{NV} x_{ij}^{v} c_{ij} + w_2 \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{v=1}^{NV} x_{ij}^{v} R_{ij}\} \qquad (1)$$

Subject to:

$$\sum_{i=1}^{n}\sum_{v=1}^{NV} x_{ij}^{v} = 1 \qquad j = M+1,...,n \qquad (2)$$

$$\sum_{j=1}^{n}\sum_{v=1}^{NV} x_{ij}^{v} = 1 \qquad i = M+1,...,n \qquad (3)$$

$$\sum_{i=1}^{n} x_{ip}^{v} + \sum_{j=1}^{n} x_{pj}^{v} = 0 \qquad v = 1,...,NV, \quad p = 1,....,n \qquad (4)$$

$$\sum_{i=1}^{n} d_i (\sum_{j=1}^{n} x_{ij}^v) \le K_v \qquad v = 1,..., NV \tag{5}$$

$$x_{ij}^v = 1 \quad \Rightarrow \quad T_i + s_i + t_{ij} \le T_j \qquad i = 1,..., n, \quad j = M+1,..., n \tag{6}$$

$$a_i \le T_i \le b_i \qquad i = M+1,..., n \tag{7}$$

$$\sum_{i=1}^{2} w_i = 1 \text{ and } w_i \in [0,1]$$

The objective function (1) expresses the weighted sum of the two objective functions. Constraints (2) and (3) imply that each point of demand is visited only once by only one vehicle. Constraint (4) implies that the vehicles do not end their routes on the point of demand. Constraint (5) implies that the quantity delivered by each vehicle at each route should not exceed its capacity. Constraint (6) imposes upper and lower bounds on the times that the vehicles may arrive at each point of demand. Furthermore, equations (7) assures that if a vehicle moves from demand point (I) to demand point (j) the time it needs to get at j is at least the service time at (i) plus the travel time from (i) to (j).

The solution method that is proposed for this problem belongs to the construction heuristic algorithms. Two alternative versions of the proposed algorithm already exist. The first one solves the single criterion vehicle routing and has been implemented and applied on the Solomon's benchmark problem sets providing encouraging results in comparison to the other existing construction algorithms. A slight modification of the first version of the proposed heuristic algorithm lead to the second version that is best suited for solving bi-objective vehicle routing and scheduling problems.

The basic steps of the proposed algorithmic approach are the following:

1. Insertion of a new customer in a new route. In this step the construction of a new route is initiated.
2. If all customers are routed then the process terminates. Otherwise for every customer (both routed and unrouted) identify its best insertion position to the already existing routes. The best insertion position is defined on the basis of the minimization of the following metric:

   $$c_1(i,u,j) = a_1(d_{i,u} + d_{u,j} - d_{ij}) + a_2(b_{j_u} - b_j) \qquad a_1, a_2 \in [0,1]$$

   Where d_{ij} is the travel time from i to j, and b_i is the service start time at customer I, and α_1, α_2 are weighting factors.
3. Selection of the new customer to be inserted.

4. Insertion of the selected customer, update the service start times of each routed customer and return to step 2.

The structure of the proposed algorithm resembles the Insertion Heuristic (I2) algorithm developed by Solomon, however it has many different features that secures its innovation. The basic difference between Solomon's I2 algorithm and proposed insertion heuristic lies in the fact that the new one considers for insertion both routed and unrouted customers while the I2 focuses on the insertion of unrouted customers only. This feature allows for reinserting already routed customers to positions that improve the objective function and consequently correcting sub-optimal previous insertions.

The proposed algorithm has been applied to the Solomon's benchmark problems that consist of several representative single objective vehicle routing and scheduling problems. The results of this application are presented in Table 1. In the same table it is also presented the respective results of other competitive construction algorithms [Solomon, 1987]. The comparison of the results of the alternative algorithms with the results of the proposed heuristic is quite encouraging for the performance of the new algorithm.

	Sweep				**I_1**				**I_2**			
	NV	TT	ST	WT	NV	TT	ST	WT	NV	TT	ST	WT
R1	14.6	1449.7	2817.4	317.7	13.6	1436.7	2695.5	258.8	14.5	1638.7	2888.1	249.4
R2	3.2	1448.6	2590.1	141.4	3.3	1402.4	2578.1	175.6	3.3	1470.7	2645.8	175.1
C1	10.0	940.8	10133.8	193.0	10.0	951.9	10104.2	152.3	10.1	1049.8	10174.3	124.5
C2	3.0	711.9	9755.2	43.4	3.1	692.7	9921.4	228.6	3.4	921.5	10151.4	229.9
RC1	14.9	1804.5	3094.5	290.0	13.5	1596.5	2775.0	178.5	14.2	1874.4	3029.5	155.1
RC2	4.0	1735.7	3007.9	272.1	3.9	1682.1	2955.4	273.2	4.1	1797.6	3128.4	330.7

	NN				**NEW-I_2**			
	NV	TT	ST	WT	NV	TT	ST	WT
R1	14.5	1600.1	2968.1	368.0	13,6	1379,7	188,1	188,1
R2	3.4	1472.3	2719.6	247.4	3,2	1280	2535,4	228
C1	10.2	1171.2	10472.5	301.3	10,4	1094,9	10290	195,5
C2	3.5	963.1	10785.9	822.7	3,4	804,6	9890,8	86,2
RC1	14.2	1800.0	3057.2	257.2	13,6	1591,3	178,6	2770,0
RC2	3.9	1754.7	3120.0	272.1	3,6	1571,2	2831,9	260,8

Table 1: The results of the application of the heuristic algorithms Sweep, I1,I2 by Solomon, Nearest Neighbour (NN) , and the proposed New I2 on the R1, R2, C1, C2, RC1, RC2 problems.

NV: average number of vehicles identified per problem category

TT: average total travel time found per problem category

ST: average schedule time per problem category

WT: average waiting time per problem category

A real life distribution problem that can be modeled as a bi-objective vehicle routing and scheduling problem is the distribution of Hazardous Materials. Hazardous Materials transportation raises concerns about the safety of the human and natural environment. Although hazardous material accidents are rare events i.e. the accident probability is extremely low, their impacts can be severe for the human and natural environment. This basic feature of hazardous materials transportation adds a special concern in the decision making process of the Hazardous Materials Logistics operations since the minimization of risk should also be taken into account [Zografos, Davis, 1989],[List et al , 1991]. The risk imposed by the transportation of Hazardous Materials is expressed by the product of the probability of an accident times measure of its consequences. Therefore, the Risk minimization in Hazardous Materials Minimization can be accomplished both by minimizing the Hazardous Materials accident probability and the reduction of the accident's impacts. The consideration of risk as a criterion for selecting Hazardous Materials distribution routes contributes substantially to the reduction of accident probability and the severity of the accident's impacts.

The second version of the algorithm has been implemented for solving the large-scale distribution problems of Petroleum and Liquefied Petroleum Products (LPG) in the Thriasion Field at the Attica Region of Greece. For this purpose the proposed heuristic algorithm was integrated within the Decision Support System for Hazardous Materials Logistical Operations [Zografos, Androutsopoulos, 2000]. The proposed Decision Support System was used to study the risk-cost trade-off for the distribution of petroleum and LPG products for several large-scale real life distribution problems. The application of the proposed algorithm gave encouraging results in terms of accuracy and speed.

AKNOWLEDGEMENTS

The reported research has been supported by the Greek General Secretariat for Science and Technology, through the funding of the project "Decision Support System for the Distribution of Hazardous Materials".

REFERENCES

[Desrochers et al, 1990]
M. Desrochers, J. Lenstra, M. Savelsbergh, "A Classification Scheme for Vehicle Routing and Scheduling Problems", European Journal of Operational Research, Vol 46, pp.322-332, 1990.

[Laporte et al, 1992]
G. Laporte, F. Louveaux, H. Mercure, "The Vehicle Routing Problem with stochastic Travel Times", Transportation Science Vol. 26, No 3, 1992.

[List et al, 1991]
List, G.F., P.B.Mirchandani, M. Turnquist and K.G. Zografos, "Modeling and Analysis for Hazardous Materials Transportation: Risk Analysis, Routing/Scheduling and Facility Location", Transportation Science, Vol. 25, No.2, May 1991, pp. 100-114

[Solomon, 1987]
Solomon, M. "Algorithms for the Vehicle Routing and Scheduling Problems with the Time Window Constraints", Operations Research, Vol. 35, 1987.

[Zografos, Androutsopoulos, 2000]
Zografos K.G., Androutsopoulos, K.N., "A Decision Support System For the Distribution of Hazardous Materials: The Case of Petroleum and LPG Products", Proceedings of the EUROLOG conference, 2000.

[Zografos,Davis, 1989]
Zografos, G. K., Davis, C.F. "Multi-Objective Programming Approach for Routing Hazardous Materials", Journal of Transportation Engineering, vol. 115, No 6, ASCE, 1989