RESUPPLY STRATEGIES FOR GENERAL TWO-ECHELON, MULTI-INDENTURE MAINTENANCE SYSTEMS
(extended abstract)

W.H.M. Zijm[1], G.J. van Houtum[2] and W.D. Rustenburg[3]

Abstract. In this paper, we consider technical systems that consist of assemblies, subassemblies and parts which may be subject to breakdowns. Upon failure of a part or component, it is replaced by a ready-for use one. The failed part is either repaired or, in case repair is not possible, it is disposed of and a new one is procured (if budget allows). Of course, replacement of a failed part is only possible if a sufficient number of spare parts is available. The *initial supply problem* focuses on the optimal allocation of a given budget for investment in spare parts stocks, such that an optimal technical system availability can be guaranteed. Here, it is assumed that failed parts can always be repaired. In reality however, condemnation may occur, leading to the need to acquire new parts from an external supplier to maintain spare parts inventory levels. To this end, the responsible maintenance facilities usually have a fixed annual resupply budget available. The question now is how to spend this annual resupply budget throughout the year, i.e. which items should be procured when, in order to optimize system availability throughout the year. In a previous paper, we discussed strategies for a simple version of the resupply problem, viz. for a single-indenture single echelon system, while further total condemnation and fixed and equal procurement lead items for all items were assumed. The current paper relaxes all these assumptions. The extensions subsequently discussed include: the situation of mixed procurement and repair, product-dependent procurement lead times, stocks at multiple (both local and central) sites (hence a two-echelon structure) and finally multi-indenture systems. This research is motivated by a series of studies carried out for, and in co-operation with, the Royal Netherlands Navy.

1. Multi-echelon spare parts management for multi-indenture technical systems

First, we describe the technical background of the systems we study in this paper. Consider a complex technical system, e.g. an installation in a process industry, an aircraft, a frigate, a nuclear power installation, for which it is natural to pursue a high system availability. When studying the system configuration, it is natural to find a hierarchical structure similar to the product breakdown structure (the *Bill of Materials*) for an industrial product. In such a system breakdown structure, we often distinguish assemblies, subassemblies, components, parts and so on (referring to the various levels in the breakdown structure. When an important part of the technical system malfunctions, the system may shut down. Next, a maintenance and repair

[1] University of Twente, Faculty of Mathematical Sciences, p.o. Box 217, 7500 AE Enschede, the Netherlands, *corresponding author*
[2] Eindhoven University of Technology, Faculty of Technology Management, p.o. Box 513, 5600 MB Eindhoven, the Netherlands
[3] Districon B.V., p.o. Box 188, 3600 AD Maarssen, the Netherlands

facility attempts to detect the assembly, subassembly or part that causes the problem, and subsequently replaces it by a ready-for-use item, in order to let the technical system run again. The failed item is repaired in principle and, upon completion, put in stock as a ready-for-use item again. The term "multi-indenture" refers to the fact that we discern a multi-level system structure, and that replacements may in principle take place at any level in that structure. Hence, one may decide to replace a complete assembly, for instance because it is much easier to do that than instead to perform more time-consuming disassembly and re-assembly operations. If in addition, multiple technical systems are operating at different locations, one may additionally hold stocks of spare parts and/or assemblies both close to each technical system and at a central repair and maintenance facility. The latter allows one to allocate items at the last moment to the system for which they are needed, however often at the expense of longer transport times. An intermediate solution is the one in which stocks are held at a central location, to supply more limited stocks that are kept close to each system installation. In this case, we speak of a multi-echelon spare parts inventory system.

2. The initial supply problem for multi-indenture, multi-echelon structures

The *initial supply problem* basically concerns the question how many parts, subassemblies or assemblies to store at the local base stock centers assigned to each technical system, and at the central depot, given a limited available budget for the initial procurement of spare items. This problem alone constitutes already a formidable intellectual challenge. First of all, it will be clear that the problem highly depends on the complexity of the breakdown structure of the technical systems. Holding complete assemblies in stock is an expensive option, but it helps to reduce system unavailability to a minimum. In addition, the failure rates and the repair capacities (or the repair lead times) at the various facilities will have a significant impact. The occurrence of parts commonality (i.e. parts that are installed in several systems, or even in different assemblies of one system) complicates the analysis considerably. Finally, failure behavior of components itself may be rather complex.

In the initial supply problem we assume that any failed item can be repaired in principle. Under the assumption of unlimited repair capacity and Poisson failure rates, this problem has been studied extensively by Sherbrooke [1], who was motivated by a series of problems encountered the American Air Force. Sherbrooke was able to describe the availability of a technical system as a non-linear function of the expected number of backorders of all items that constitute this technical system (which in turn depend on the base stock levels). In addition, exploiting certain convexity properties of the logarithm of this function, he showed that a greedy procedure yields optimal or near-optimal stock allocations with respect to all parts and all possible locations (here optimality refers to the maximum system availability). One of the consequences of this approach is the intuitively appealing result that it is better to have ample stock of cheap items and to be more careful with investing in expensive items. That is the "biggest bang for the buck" approach in which items (assemblies, subassemblies or parts) are ranked according to their contribution to decrease the expected backlog per unit of money invested. Hence, in case of equal failure rates and repair lead times, the base stock of a cheap item will in general be higher than that of an expensive item.

The models discussed by Sherbrooke became known as METRIC models (Multi-Echelon Technique for Recoverable Item Control). A number of extensions were developed by Muckstadt [2], Slay [3], Graves[4] and again Sherbrooke [5], [6], leading to models such as MOD-METRIC and VARI-METRIC. All these systems assume in principle unlimited repair capacities; extensions to limited repair capacities are discussed by e.g. Gross et al. [7], Albright [8], Diaz and Fu [9] and Avsar and Zijm [10]. Nevertheless, these models show a number of serious shortcomings when applied to real-life technical system, as well as to the structure of many repair and distribution structures. Rustenburg et al. [11] discuss a number of these shortcomings in detail. In the first six chapters of his dissertation, Rustenburg [12] extends the analysis of the initial supply problem substantially, including the generalization to non-Poisson failure processes, as well as methods to deal with redundancy, commonality and criticality of subsystems. As in Sherbrooke [6], an important role is played by the so-called *availability-investment trade-off curves* that relate spare parts inventory investments to a maximum attainable system availability. Associated with each point on these trade-off curves are base-stock levels for all items (parts, subassemblies and assemblies) that together constitute the system installations. All these models have been applied to real-life examples at the Royal Netherlands Navy, with very satisfactory results, i.e. a significant reduction of investments while at the same time a significant increase in system availability. The results of these case studies will be discussed in a forthcoming paper.

3. Re-supply problems during the system's exploitation period

A serious drawback of the METRIC-type models developed so far is the fact that they are static in nature, i.e. they assume a stationary stochastic failure behavior, no condemnation (items can always be repaired), and a never changing system structure. Unfortunately, such assumptions are seldom realistic. First, due to technological progress and the implementation of modifications as a result of earlier maintenance events, an organization will usually attempt to reduce the failure rates. Further, condemnation does occur and new items have to be procured (and again the failure rates may change). The latter is called *resupply*, as opposed to the *initial supply* discussed above. Unfortunately, the annual budgets available for purchasing new components may not be sufficient to replace all condemned items, or, equivalently, to keep up the operational availability level as determined by the initial investment. This causes the need to rethink the availability level and the desired levels for the inventory positions. Also, being confronted with sometimes even declining budgets for resupply, a company may wonder what will be the best strategy to spend the available budget during a certain year, in order to attain a maximum expected average availability over that year.

In an earlier paper, we have studied these resupply problems for a rather simple situation (cf. Rustenburg et al. [13]), In particular, we considered a single-echelon, single indenture structure in which all failed items are condemned while the procurement lead times of the various items are identical and deterministic. Under these assumptions we have shown that a so-called Balance Focussed strategy yields highly satisfactory results. In words, this strategy works as follows: at any occurrence of a failure, or possibly at frequent regular time instants, we estimate the number of items to be procured during the remaining budget year, and compare that with the available budget. In case the latter is insufficient to cover all expected procurement costs, we determine the highest point on the *availability-investment trade-off curve* such that the corresponding set of base stock levels for all items still can be attained given the limited remaining budget (based upon existing failure rates and the probabilities that failed items have to be condemned).

In the current paper, we extend the analysis of the resupply problem and in particular of the Balance Focussed strategy to situations with mixed repair and condemnation, with product-dependent procurement lead times, and with maintenance systems with two-echelon and multi-indenture characteristics. First we show that also in the case where items are either repaired or disposed of, the availability of a system at some time t depends on the inventory positions of its constituting items at different earlier time instants, namely the inventory position of item i at time $t-T_{P(i)}$ where $T_{P(i)}$ denotes the procurement lead time of item i. Note that the repair lead times do not play any role in the determination of the relation between the system availability and previous item inventory positions. Another complication arises from the fact that usually procured items are paid only after delivery. In case all procurement lead times are equal, this simply means a shift in expenses; with different procurement lead times however we have to take into account the effects of current decisions on future (next year's) budgets. Finally, we discuss extensions of our models to situations where we may have both local and central stocks, and to more complicated systems that reveal a multi-indenture system structure. Numerical results for small examples are presented; in addition, we evaluate more extensive case studies carried out for three major technical systems aboard of the M-frigates of the Royal Netherlands Navy, i.e. a long range air surveillance radar system, a diesel engine system for the ship's propulsion, and the Goalkeeper air missile defense system. Finally, we discuss further extensions and future work.

References:

1. SHERBROOKE, C.C. [1968], METRIC: A Multi-Echelon Technique for Recoverable Item Control, *Operations Research*, Vol. 16, pp. 122-141.
2. MUCKSTADT, J. [1973], A model for a multi-echelon, multi-indenture inventory system, *Management Science*, Vol. 20, pp. 472-481.
3. SLAY, F.M. [1984], VARI-METRIC: An approach to modelling multi-echelon resupply when the demand process is Poisoon with a Gamma prior, Technical Report AF301-3, Logistics Management Institute, Washington D.C.
4. GRAVES, S.C. [1985], A multi-echelon inventory model for a repairable item with one-for-one replenishment, *Management Science,* Vol. 31, pp. 1247-1256.
5. SHERBROOKE, C.C. [1986], VARI-METRIC: Improved approximations for multi-indenture, multi-echelon availability Models, *Operations Research*, Vol. 34, pp. 311-319.
6. SHERBROOKE, C.C. [1992], *Optimal Inventory Modeling of Systems: Multi-Echelon Techniques*, Wiley, New York.
7. GROSS, D, D.R. MILLER, AND R.M. SOLAND [1983], A closed queueing network model for multi-echelon repairable item provisioning, *IIE Transactions*, Vol. 33 (11), pp. 1453-1468.
8. ALBRIGHT, S.C. [1989], An approximation to the stationary distribution of a multi-echelon repairable item inventory system with finite sources and repair channels, *Naval Research Logistics*, Vol. 9, pp. 179-195.
9. DÍAZ, A., AND M.C. FU [1973], Models for multi-echelon repairable item inventory systems with limited repair capacity, *European Journal of Operations Research*, Vol. 97, pp. 480-492.
10. AVSAR , Z.M., AND W.H.M. ZIJM [2000], Resource-constrained two-echelon inventory models for repairable item systems, Memorandum 1521, Faculty of Mathematical Sciences, University of Twente, accepted for publication.
11. RUSTENBURG, W.D., G.J. VAN HOUTUM AND W.H.M. ZIJM [2000 A], Spare parts management at complex technology based organizations: An agenda for research, Accepted for publication in *The international Journal for Production Economics*.
12. RUSTENBURG, W.D. [2000], *A System Approach to Budget-Constrained Spare Parts Management*, PhD Thesis, Eindhoven University of Technology, Eindhoven.
13. RUSTENBURG, W.D., G.J. VAN HOUTUM AND W.H.M. ZIJM [2000 B], Spare parts management for complex technical systems: Resupply of spares under limited budgets, Accepted for publication in *IIE Transactions*.