INVENTORY POSITIONING IN SINGLE PRODUCT SUPPLY CHAINS

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Abstract:

The new economy introduces significant pressures on supply chain participants, calling for constant inventory reductions and capital investments only in value adding / revenue-generating assets. A problem that arises due to these intense pressures is that of postponing, i.e., pushing inventory holding to low value locations. This paper addresses inventory positioning, i.e., the concurrent determination of: a) the supply chain node where inventory should be held, and b) the appropriate inventory level. The objective is to minimize holding costs, given a pre-specified order fill rate. A single-echelon inventory system models the problem. Inventory value is assumed an increasing function of the amount of processing performed at upstream nodes, while achieved fill-rates are considered a function of the distance-time between the inventory and customer locations. We propose a novel, analytical and simple approach for the case of normally distributed demand and single end-item supply chains that is based on iterative calculations of inventory holding costs at the various potential inventory locations. An example illustrates the mechanics of the approach in practical situations.

***Keywords*:** Inventory Management, Supply Chain Design

1. Introduction

Supply chain management (SCM) made its appearance in the literature over the last decade, expanding the notion of logistics. In the era of the "new economy" it seems to attract significant management attention, viewed as a fundamental strategic concept and an opportunity to gain competitive advantage. In simple terms, SCM is making sure you have the right product, in the right place at the right time and at the right cost.

In the emerging information society the concepts of time and space have become even more crucial to the managerial decision-making. Product life cycles are reduced dramatically, customers are more sensitive to delivery times and service quality, demand for highly customized goods is increasing, information collection and comparison of products as far as both prices and quality are concerned require less effort and are less time consuming, making consumers less loyal. Moreover, competition is taking place in a global environment. As these changes and challenges become apparent managers are forced to alter their supply chain perspective and adopt an inter-organizational approach instead of focusing on the individual firm. The goals of the entire supply chain become the common objectives of each firm. (Ernst and Kamrad, 2000). Hopefully a Pareto optimum solution will exist where all the players will be better off. The Internet and the new technology enablers can materialize this new vision.

In the context of supply chain management, under the pressure of globalization, mass customization and of reducing inventory costs, the notion of postponement seems to attract increasing interest. Postponement is an organizational concept whereby some of the activities in the supply chain are not performed until customer orders are received opposed to producing in anticipation of future orders. Postponement can occur along the entire supply chain from sourcing to final distribution (van Hoek, 2001). Bowersox and Closs (1996), van Hoek (1998), van Hoek (1999) mention three types of postponement: time postponement, when activities are postponed until orders are received in time; place postponement, when inventories are kept upstream in central manufacturing or distribution centers until orders are received; form postponement, when activities that determine the form and functions of products are delayed until orders are received. Manufacturing postponement combines all three types of postponement. Final processing and manufacturing are postponed (time postponement), are performed upstream in the supply chain, inventories are kept centrally (place postponement) and products are delayed from taking their final customer and country specific attributes (form postponement) until orders are received.

Alderson (1950) introduced the concept of postponement observing that products tend to become differentiated as they reach the end of the supply chain. Bucklin (1965) investigated the positioning of inventory in the marketing channel. In the eighties Shapiro (1984) examined different degrees of postponement applications as an answer to lower inventory cost, facilitate product innovation and improve customer service.

In the 90's Lee and Billington (1992), by examining the common pitfalls in viewing the supply chain management, proposed that companies under the weigh of increased demand for highly diversified products and reduced product life cycles should focus primarily on design for supply chain than design for manufacturability. In modeling postponement Lee *et al*. (1993) simulated Hewlett-Packard's supply chain for a specific printer model concentrating on inventory levels in order to compare two alternative strategies for postponing. They conclude that for the specific case, postponing customization and keeping inventories of semi-finished goods at a central site, reduced significantly the total inventory investment, saving the company millions. Davis (1993) reached similar results through several cases from Hewlett-Packard's product families, whose manufacturing has been reengineered from a supply chain's perspective.

Feitzinger and Lee (1997) addressed the problem of increased demand for highly customized products and elevated service rates at acceptable cost levels by marrying the notions of postponement and modularization (assembly of products from standardized constituent units). Ernst and Kamrad (2000) categorized different structures of supply chain according to postponement and modularization and illustrated, by numerical examples, the total cost of employing a particular supply chain formation. They emphasized the notion that combining modularization and postponement is more profitable than using the two functions separately. Van Hoek (1999) examined the implementation of postponement in the food, electronics, automotive, and clothing industry, in relation to outsourcing and spatial reconfiguration of the supply chain. He concluded that food companies apply postponement at a lower level than companies in other industries and drew attention to the possibility of linking outsourcing customized activities and postponement. Furthermore, he deduced that postponement requires a spatial reconfiguration of the supply chain. Through a survey and a case study the same author (1998) highlighted postponement as a key element within the virtual integration business model, while in (2001) he presented the taxonomy of existing literature on postponement.

The problem we address in this paper emanates from the postponement literature, but offers a novel, simple modeling approach that can assist managers in the design of supply chains with multiple nodes. Specifically, we consider single product supply chains and we determine the node in which inventory should be held in order to minimize inventory holding costs under service level constraints. The problem is modeled as a single-echelon inventory system, where times and values are the result of inter-nodal transportation and node processing, respectively. Demand is driven by information from point-of-sales and is assumed normally distributed, while fill-rates are considered a function of the distance-time between the inventory and customer locations. We propose a solution methodology based on iterative calculations of inventory holding costs at the various potential inventory locations, which account for inventory quantities specified by appropriate loss-function evaluations. Through an example, we demonstrate the applicability of the approach in practical situations.

The remainder of the paper is organized as follows. Section 2 defines in details the problem under consideration. Section 3 describes the proposed optimization model, while Section 5 outlines the developed solution approach. Section 4 presents an illustrative example. The paper concludes in Section 5.

2. Problem Definition

We consider a typical supply chain with multiple nodes that are interconnected via fixed transportation paths. Each node of the supply chain is a distinct processing location. By processing we refer to all types of value adding activities that are related to the manufacture or distribution of a product. For example, a manufacturing process that takes place in a production facility, an assembly process performed at an assembly plant, a packaging process carried out at a warehouse, or a distribution process closing the cycle at a central distribution hub, are potential processing activities related to a specific supply chain node.

The end-nodes of the supply chains considered are always Points of Sales (POS), where demand is entered into the integral system; by POS we do not refer only to consumer outlets, but to terminating nodes of supply chains that are connected to retailers.

We further constraint the problem by focusing on supply chains that are characterized by a single end item, which is the result of several value adding processes at the various nodes of the supply chain. More specifically, starting from the raw material providers, we assume that at each supply chain node, processing adds value to the upstream (incoming) material, which is then forwarded to the downstream processing locations until the POS is reached. The time that it takes to perform the value-adding activities at each node is assumed known.

To simplify the problem, we consider streamlined chains in which inventory is stored at a single location. Thus, material flows in discrete patterns from one node to a downstream node, until the inventory location is reached. We refer to this supply chain node as the buffer node, which decouples upstream and downstream processing. Appropriate inventory transfers from the buffer node to the downstream nodes fill POS demand, while no inventory is held at any other supply chain node other than the buffer.

Figure 1 illustrates a representative supply chain. Each arrow in Figure 1 denotes the transportation of products from one node to the subsequent one, while circles represent supply chain nodes; B is the buffer node where inventory is held.

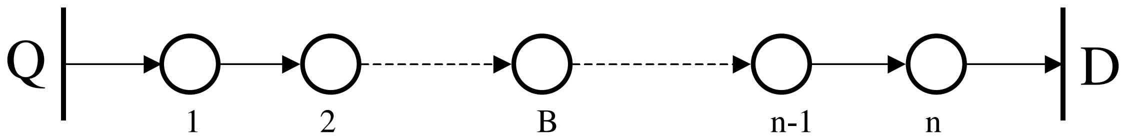

Figure 1: Representation of a Supply Chain

The transferring of goods between nodes takes a certain amount of time that is related to the distance from the upstream to the downstream node, the mode of transportation and the specific characteristics of the product. For each route we can only have an approximation of the time needed for the transportation, since an element of uncertainty is implied. Uncertainty may vary according to distance, mode of transportation, weather, traffic, seasonal factors, customs formalities and other unexpected incidents such as strikes, accidents.

As products go through a process their value increases and this is intensified as we proceed to nodes near the POS. Thus, at the supply chain level, holding inventory at downstream nodes requires tying more capital to non-revenue generating items (inventory). Consequently, at the design stage of supply chains, management should allow inventory-holding locations only at the beginning of the chain, if inventory holding cost minimization is the only objective. On the other hand a key managerial goal is to reach very high levels of service for the end customers, which can be translated into high order fill and customer response rates. These two measures depend upon the time required to reach demand from inventory holding locations, which includes the transportation time between nodes and the processing time at the buffer-to-POS nodes. It is evident, that keeping inventory near the POS will result in the maximization of fill and response rates.

Balancing the above two objectives must be achieved through a concurrent approach that determines the location of the buffer and the inventory quantity stored at the buffer node, given certain target service levels imposed by management and demand patterns induced by customers. This is exactly the problem we address in this paper, considering a single time period in which products either reach the buffer node or fill end-customer orders.

3. Model Description

The problem we consider can be stated as follows: Find the node where products should be stocked in order to fill a certain level of demand with minimum holding cost. The number of supply chain nodes (procedures) is n, and the number of routes connecting the nodes is n-1. Indices i and j refer to nodes, and take values between 1 and n, while ordered pairs of these indices refer to routes.

There is a quantity Q expressed in units of a product that enter the system from node i=1 at the beginning of the period; some or all of these units exit from node i=n at the end of the period, after passing through all the intermediate nodes of the supply chain. Q is a decision variable for our problem. Furthermore, there is a demand D, which is modelled by a continuous nonnegative random variable, with mean μ and standard deviation σ.

Three different parameters are assigned to each node:

a. An inventory value-added c_i, which represents the increase in the value of the product after it has completed the process at node i, compared to the value it had when entering this node.
b. The quantity stored at each node q_i, which is zero for all nodes except the buffer node.
c. An extra percentage of the product that could be delivered to the customer, because the buffer is closer to the end node of the supply chain ϕ_i.

The inventory values added at all the nodes before the buffer node, sums up to the inventory holding cost per unit c_h at the buffer locations.

$$c_h = \sum_{i=1}^{B} c_i \tag{1}$$

The quantity stored at the buffer, if this is node i of the supply chain, is defined as follows:

$$q_i = \begin{cases} \text{Q, if i is the buffer building node} \\ \text{0, if i is not the buffer building node} \end{cases} \tag{2}$$

Thus, the sum of the units stocked at each buffer is the quantity produced.

$$Q = \sum_{i=1}^{n} q_i \tag{3}$$

We assume that there are no losses while transporting products from the initial node to the buffer (or equivalently, that the losses are incorporated in the quantity that enters the system, which may be expressed in units of products that can reach the buffer). In addition, $\sum_{i=1}^{j} \phi_i$ is the probability that the product will reach the end customer if buffer is located at node $j=B$. We assume that this probability is known and calculated using the following time-related equation:

$$\sum_{i=1}^{j}\phi_i = \frac{\sum_{i=1}^{j} pt_i + \sum_{i=1}^{j-1} tt_{i,i+1}}{\sum_{i=1}^{n} pt_i + \sum_{i=1}^{n-1} tt_{i,i+1}} \times \sum_{i=1}^{j}\alpha_i \tag{4}$$

In (4), pt_i is the processing time at node i, $tt_{i,i+1}$ is the transportation time from node i to node $i+1$ and α_i is a factor relevant to anything unexpected that could happen, while transferring the goods from node i to node $i+1$, and delay the delivery of the product (e.g., weather, strikes, increased seasonal traffic, delay in customs, etc.). Factor α_i can differ according to the mode of transportation, the time of the year, the distance between the nodes, the country where the node is located, etc.

From the definitions of q_i and ϕ_i we can identify the quantity delivered to the customer, denoted by q. This may be different from the quantity produced Q; q will only be a proportion of the initial quantity that enters the system, since a part of it may be lost during processing or transportation from one node to another, or could not reach the end customer on time.

$$q = \sum_{j=1}^{n}\left(\sum_{i=1}^{j}\phi_i\right)q_j \tag{5}$$

It should be noted that time is not stated explicitly in the model but is implied through the definition of q and ϕ. Finally, management (or market constraints) imposes a desired service level, β. This parameter may also be described as the proportion of demand that we wish to be covered from stock, or as the probability that demand would be met; β takes prices from the closed set [0,1].

4. Solution Approach

To solve our problem, we need to follow the steps that are listed below:

1. Develop an expression for the holding cost as a function of q_i.
2. Develop an expression for the number of shortages that will occur in one period as a function of both the random variable D and the decision variable Q.
3. Determine the value of Q and the holding cost if buffer is placed at node $i=1$. Repeat the calculations for the case that the buffer located at nodes $i=2$ to $i=n$.
4. Determine the optimal quantity to be produced, which will be the one corresponding to the lower possible holding cost.

In the following, we expand on all the above steps and propose our specific approach.

Development of the inventory holding cost function

The expected inventory level will be the quantity that enters the system; hence it will be equal with the decision variable Q. At each node the holding cost will be the holding cost per unit of inventory (c_h) times the quantity stored in that node (q_j). For all the nodes except the buffer, the quantity q_j ($j \neq B$) will be zero; therefore the holding cost will be zero as well. For the buffer building node $j=B$, the inventory holding cost will be the holding cost per unit times the quantity stored at the buffer q_B which is equal with the quantity that entered the system Q. Consequently, the holding cost function will be:

$$C(Q) = (\sum_{i=1}^{B} c_i)Q = (\sum_{i=1}^{B} c_i)q_B = \sum_{j=1}^{n}(\sum_{i=1}^{j} c_i)q_j \tag{6}$$

Expected number of stock-outs

Stock-outs occur for two reasons: a) If demand exceeds production (D>Q); b) inability to cover the demand when the buffer is too far from the end-customer. In the latter case, the product may not reach the end customer because of long transportation times or unexpected factors that are more likely to happen the longer the products have to travel. Since the demand is normally distributed, it follows that the expected number of shortages that occur in one period is given by the expression

$$E(\max(D-q,0)) = \int_0^{\infty}(x-q)f(x)dx \tag{7}$$

where $f(x)$ is the probability density function.

The greater value for q is reached when $\sum_{i=1}^{j}\phi_i$ =1, i.e., the quantity delivered is equal to the quantity stored, which is equal with the quantity produced, $q=Q$. Therefore, given Equation (5), (7) can also be expressed as:

$$E(\max(D-q,0)) = \int_Q^{\infty}(x-\sum_{j=1}^{n}(\sum_{i=1}^{j}\phi_i)q_j)f(x)dx \tag{8}$$

Equation (8) defines the parameter $S(q)$, which is the expected number of shortages. As the expected demand per period is μ, it follows that the proportion of demand that is not met on time is:

$$(1-\beta) = \frac{\text{Expected Number of Shortages per period}}{\text{Expected Demand per period}} = \frac{S(q)}{\mu} \tag{9}$$

To determine $S(q)$, since D is normally distributed, we can use the normal loss function, $NL(z)$. Specifically, the expected number of shortages that will occur during one period is $\sigma NL(z)$ (Winston, 1997; Nahmias, 1997); i.e., if we hold quantity that equals the average demand and z standard deviations ($\mu+z\sigma$) then $\sigma NL(z)$ is the expected number of shortages during one period. Since delivering a larger quantity will lead to fewer shortages, we would expect $NL(z)$ to be a non-increasing function of z. Intuitively, this must be true in view of the fact that as the quantity reaching the buffer node increases (which leads to an increase in the quantity delivered to the customer) the expected number of shortages must decrease. Moreover, as the probability of delivering the products on time (i.e., $\sum_{i=1}^{j}\phi_i$ for increasing j) increases (equivalently, as the buffer is located closer to the end of the supply chain, which leads to an increase in the quantity delivered to the customer) $\sigma NL(z)$ is expected to decrease. From (9) we have that $S(q)$=(1-β)μ, thus we can calculated the expected number of shortages. Since $S(q)$= $\sigma NL(z)$, $NL(z)$ can be directly calculated, and then z can be found from appropriate tables (Winston, 1997; Nahmias, 1997). Given that $q=\mu+z\sigma$, the quantity delivered to the end customer, can then be calculated.

Given the quantity delivered at the end customer and the probability of delivering on time, we can determine the quantity reaching the buffer *B* for the different positions of the buffer in the supply chain and the corresponding holding costs, which are different for each buffer location. The goal is to choose the buffer location and the corresponding inventory that leads to the smaller holding cost, while fill rates are met.

5. Numerical Example

To illustrate the model and the approach developed in Sections 3 and 4, we consider the supply chain of Figure 2. A plant situated at a European country is processing the raw materials and produces a semi-finished product, which is transferred by tracks to another city of the same country where a manufacturing plant is located. At the plant, manufacturing and basic packaging is performed, and then the product is shipped to another country where additional de-/re-packaging takes place. Subsequently, the product is transferred to a central wholesaler, before it is forwarded to a local wholesaler. Finally the product reaches its final supply chain node, i.e., a local retailer.

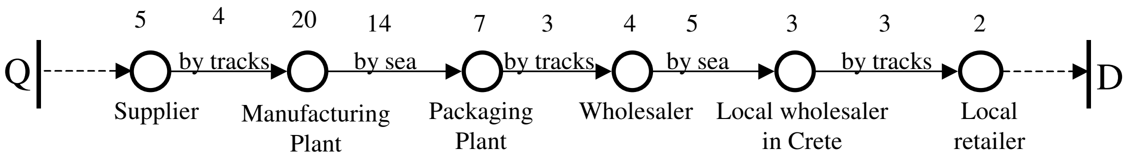

Figure 2: Supply chain of the example

In Figure 2, the numbers on top of each node represent the time that each process requires, and those above each arrow correspond to transportation times. The notes above the arrows indicate the mode of transportation, while the text below each node specifies the type of the supply chain node. Table 1 provides the values of the problem parameters.

Table 1: Values of problem parameters

Node	pt_i	$tt_{i,i+1}$	c_i	$\sum_{i=1}^{B} c_i$	$\sum_{i=1}^{B} \alpha_i$	ϕ_i	$\sum_{i=1}^{B} \phi_i$
1	5	4	5	5	0.00	0.00	0.00
2	20	14	45	50	0.90	0.38	0.38
3	7	3	50	100	0.96	0.31	0.69
4	4	5	30	130	0.97	0.10	0.79
5	3	3	30	160	0.99	0.13	0.92
6	2	0	40	200	1	0.08	1
Total	**41**	**29**					
Total Time		**70**					

Moreover, the mean of the demand is 1000 units, the standard deviation is 200 units and the target fill rate is 95%. In order to find the buffer location and the quantity that should be produced at minimum cost and for the desired service level the subsequent steps are followed:

STEP 1: Calculate the expected number of shortages per period, $S(q)$; according to mean demand and desired service level (95%), i.e., $S(q)=(1-\beta)\mu=(1-0,95)1000=50$.

STEP 2: From $S(q)$ compute the normal loss function $NL(z)$ and obtain from the tables the standardized variate z; i.e., $S(q)=\sigma NL(z)=50$. Thus, $NL(z)=0.25$, and from tables $z=0.34$.

STEP 3: Calculate the quantity delivered to satisfy the 95% fill rate; $q=z\sigma+\mu=0.34\times200+1000=1068$.

STEP 4: Calculate the quantity of inventory and the resulting holding cost if the buffer is placed at the final node $i=6$; $q=\sum_{j=1}^{n}(\sum_{i=1}^{j}\varphi_i)q_j=(\sum_{i=1}^{6}\varphi_6)q_6=1\times q_6=1068$. Then the holding cost is $q_6\times\sum_{i=1}^{6}c_i=1068\times200=213600$.

STEP 5: Repeat step 4 for buffer placed at node $i=5$, …, 1. Table 2 provides the results of these calculations for all nodes.

Table 2: Results for all supply chain nodes

Node i	Inventory Level q_i	Total Holding Cost $q_i\times\sum_{j=1}^{i}c_j$
6	1068	213600
5	1162	185883
4	1352	175779
3	1558	155750
2	2864	171862
1	-*	-*

* Cannot be calculated since $a_1=0$.

STEP 6: Select the node where the buffer should be placed and the associated inventory level (node 3 in our example).

In order to ensure the robustness of our decision, the above analysis was repeated for a higher level of fill rate, i.e., 99%. The results indicated that the solution was still to locate the buffer at node $i=3$, which means that the packaging plant should be allocated the inventory holding function.

6. Conclusions

In this paper we considered single product supply chains driven by normally distributed demand from points-of-sales. The problem we examined was a version of postponing, which frequently arises in the literature, but has mainly been tackled through qualitative methods. We proposed a novel, analytical and simple approach for determining the supply chain node in which inventory should be held in order to minimize inventory-holding costs under service level constraints. The inventory value was assumed an increasing function of the amount of processing performed at upstream supply chain nodes, while achieved fill-rates were considered a function of the distance-time between the inventory and customer locations. The mechanics of the approach were demonstrated through a simple example.

The model and the solution approach can offer significant insights for actual supply chain applications. Further research is though needed in order to extend the model to multi-product, multi-period, and multi-echelon systems, which are the majority of real-life supply chains.

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