QUEUING BASED PERFORMANCE ANALYSIS OF TANDEM QUEUES WITH PLANNED ORDER RELEASE TIMES

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Where innovation starts

AGENDA

- Introduction
- Past research
- Approximation of stage behaviour given stage lead times
- Quality of the approximation
- Conclusions

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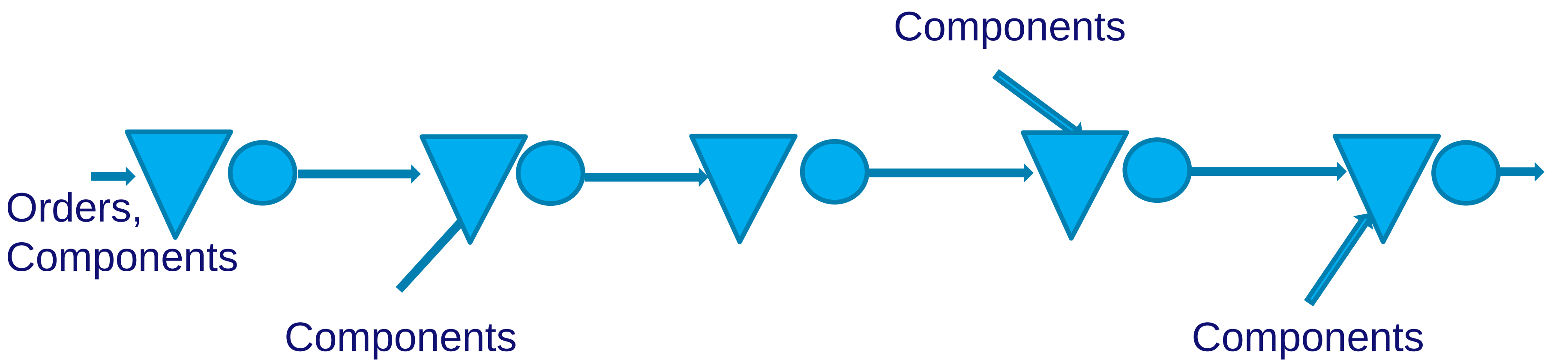
Introduction

- **Tandem queues/Serial production line:**



Introduction

- **Tandem queues**
- **Planned order release times:**
  - **At different stages expensive/superficial/…components might be needed → as late as possible (as less waitingtime as possible)**

Stage lead times (or release dates) for each order

Introduction

- **Tandem queues**
- **Planned order release times**
- **Performance analysis:**
  - **Trade – off between total lead time costs (back order costs, loss of goodwill,…) and (total) operating costs: inventory carrying costs,tardiness costs ……. at the stages**

Very long stage leadtimes might lead to no backorder costs but also lead to high inventory carrying costs etc.

Past Research

- **Atan et al., Yano, Matsuura et al., Elhafsi, Gong et al. etc.**

  Dynamic programming, recursive schemes to determine optimal planned lead times assuming a certain (empirical or theoretical) throughput time distribution at each stage

Evaluate the behaviour with planned lead times given stage throughput time distributions

Past Research

- **Not useful when (re-)designing the line, because throughput time distributions are not known at that time**

  Queuing based analysis might be of profit in the design phase since we then can investigate the impact of design variables on optimal planned lead times

  So we need a model for the stage behaviour

Approximation stage behaviour (I)

- **Tandem queue having N stages (1 machine)**
- **For each stage *i* a lead time L_i ; only when this limit has passed an order is allowed to go to the next stage -> planned release time for the next stage**
- **Negative exponential processing times at each stage**

Approximation stage behaviour (I)

- **Tandem queue having N stages (1 machine)**
- **For each stage *i* a lead time L_i**
- **Negative exponential processing times at each stage**

  -> throughput time follows a negative exponential
- **distribution: depending on interarrival time**
- **Need: distribution of the interarrival times**
- **Interarrival time at stage *n* = Interdeparture time at stage *n-1***

⇩

- ***Distribution of the interdeparture times given L_{n-1}***

Approximation stage behaviour (II)

APPROACH:

- **Decompose the line in separate production phases**
- **Approximate non-renewal processes by renewal**
- **Only consider the first two moments of arrival/ departure processes**

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Approximation stage behaviour (III)

- S_n : the throughput time of job n
- A_n : $I_{n+1} - I_n$ interarrival time of job n and $n+1$
- B_n : processing time of job n
- J_n : departure time of job n

- Then we get for the interdeparture time (n, n+1):

$$D_n = J_{n+1} - J_n =$$
$$= A_n + max(L, max(0, S_n - A_n) + B_{n+1} - max(L, S_n) =$$
$$= A_n + max(L, B_{n+1}, S_n + B_{n+1} - A_n) - max(L, S_n)$$

A, S and B mutually independent

Approximation stage behaviour (IV)

- First two moments of *D (A)*
- For i>1 approximate the interarrival time distribution at stage *i* by a mixture of two (Erlang, Exponential) distributions;
- Assume the Laplace-Stieltjes transform is $A^*(s)$
- Negative exponential processing time; μ

  then throughputtime neg. exp. distr.; $\gamma=\mu(1-\sigma)$,

  with σ unique solution of $\sigma=A^*[\mu(1-\sigma)]$ $(0<\sigma<1)$

- Given L_i the distribution of interarrival times at stage $i+1$ can be determined.

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Quality of approximation

- **4-station tandem queue; mean processing time at each station is 1**
- **Medium utilized (75%) and high utilized (95%) system**
- **Interarrival times generated by a Beta distribution and a CV of 0.5, 1.5 and 2.0**
- **Lead time L_i:**

  0

  average throughput time first station

  two times average throughput time first station

$$L_n = \text{(arrival time station 1)} + L_1 + L_2 + \ldots + L_{n-1}$$

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Quality of approximation: results 75% (A=approximation; S=simulation)

ρ	CV	L	WC 1		WC 2		WC 3		WC 4	
			A	S	A	S	A	S	A	S
0.75	0.5	0	0.5	0.5	0.87	0.85	0.96	0.95	0.98	0.97
		AT	0.5	0.5	0.64	0.63	0.73	0.71	0.79	0.76
		2*AT	0.5	0.5	0.55	0.54	0.60	0.56	0.63	0.57
	1.5	0	1.5	1.5	1.19	1.07	1.06	1.03	1.02	1.02
		AT	1.5	1.5	1.34	1.28	1.24	1.24	1.16	1.22
		2*AT	1.5	1.5	1.44	1.41	1.39	1.40	1.35	1.39
	2.0	0	2.0	2.0	1.41	1.25	1.15	1.14	1.05	1.09
		AT	2.0	2.0	1.75	1.66	1.58	1.62	1.46	1.60
		2*AT	2.0	2.0	1.90	1.86	1.82	1.85	1.76	1.85

Quality of approximation: results 95% (A=approximation; S=simulation)

ρ	CV	L	WC 1		WC 2		WC 3		WC 4	
			A	S	A	S	A	S	A	S
0.95	0.5	0	0.5	0.5	0.97	0.97	1.0	1.0	1.0	1.0
		AT	0.5	0.5	0.71	0.71	0.83	0.79	0.90	0.85
		2*AT	0.5	0.5	0.58	0.58	0.66	0.61	0.72	0.63
	1.5	0	1.5	1.5	1.05	1.01	1.00	1.00	1.00	1.00
		AT	1.5	1.5	1.33	1.32	1.23	1.29	1.17	1.27
		2*AT	1.5	1.5	1.44	1.43	1.39	1.42	1.35	1.42
	2.0	0	2.0	2.0	1.10	1.05	1.01	1.01	1.00	1.01
		AT	2.0	2.0	1.70	1.68	1.54	1.65	1.43	1.63
		2*AT	2.0	2.0	1.89	1.88	1.81	1.88	1.74	1.88

Conclusions (I)

- **The more downstream the stage, the worse the approximation**

  CV < 1: approximation values higher then simulation

  CV > 1: approximation values lower then simulation

- **The higher the utilization rate:**

  the higher CV the larger the difference

  the larger *L* the larger the difference

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Conclusions (II)

- **Is this difference bad?**
  - **Depends on the results of the cost based lead time optimization**
  - **Often: “The first blow is half the battle”**

Conclusions (II)

- **Is this difference bad?**

  - **Depends on the results of the cost based lead time optimization**

  - **Often: "The first blow is half the battle"**

So performing cost based lead time optimization experiments is now the first thing to do.

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$$A^*(s) = q(\frac{\lambda_1}{\lambda_1 + s})^k + (1-q)(\frac{\lambda_2}{\lambda_2 + s})^l.$$

$$\sigma = A^*[\mu(1-\sigma)].$$

$$x - q(\frac{\lambda_1}{\lambda_1 + \mu(1-x)})^k - (1-q)(\frac{\lambda_2}{\lambda_2 + \mu(1-x)})^l = 0.$$

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WITH CORRECTION

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