Optimal buffer allocation in serial production lines operating under Installation Buffer (IB), Echelon Buffer (EB), and CONWIP policies

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Outline

- Installation Buffer (IB) vs Echelon Buffer (EB) policy
- Performance evaluation of EB policy (sketch)
- Buffer Allocation Problem
- Solution methodology
- Numerical results on the optimal IB, EB, and CONWIP policies
- Conclusions

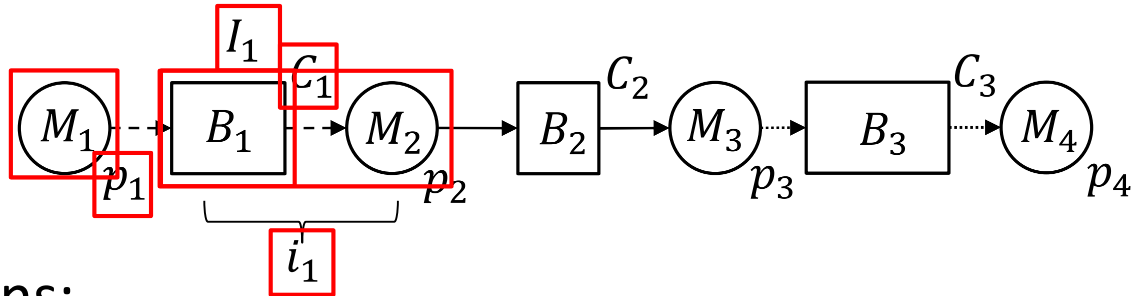
Installation Buffer (IB)

vs

Echelon Buffer (EB) policy

Installation Buffer (IB) policy

- Serial production line operated under IB policy



- Definitions:
  - M_n = machine, $n = 1, \dots, N$
  - p_n = probability that M_n produces a part in a period, $n = 1, \dots, N$
  - B_n = intermediate buffer, $n = 1, \dots, N-1$
  - C_n = intermediate buffer capacity, $n = 1, \dots, N-1$
  - I_n = installation buffer (IB), $n = 1, \dots, N-1$

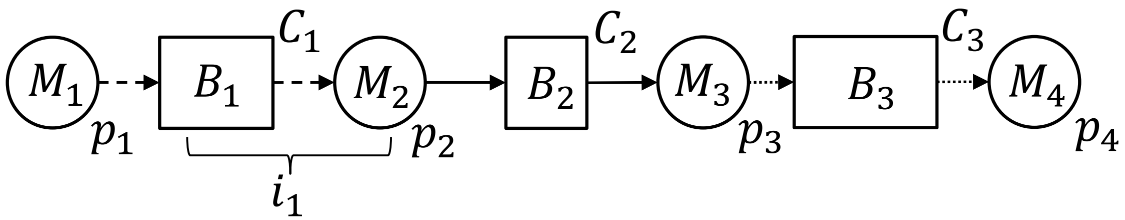
$$I_n = B_n \cup M_{n+1}, n = 1, \dots, N-1$$

  - i_n = installation WIP, $n = 1, \dots, N-1$

$$i_n \leq 1 + C_n \equiv \text{ capacity of } I_n$$

IB policy

- Serial production line operated under IB policy



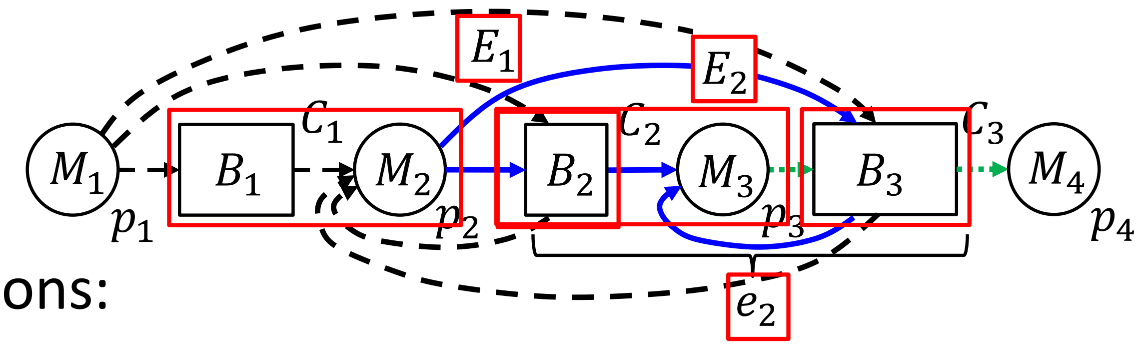
- Operation:
  - Machine M_n is allowed to store the parts that it produces in its immediate downstream buffer B_n if the next machine M_{n+1} is occupied, $n = 1, \dots, N-1$.
  - Machine M_n is blocked before service from processing a part if the number of parts that have been produced by it but have not yet departed from the next machine M_{n+1} is equal to $1 + C_n$, i.e.,

$$i_n = 1 + C_n$$

Is there a way to increase the utilization of buffer space?

Echelon Buffer (EB) policy

- Serial production line operated under EB policy



- Definitions:
  - E_n = Echelon buffer (EB), $n = 1, \dots, N-1$

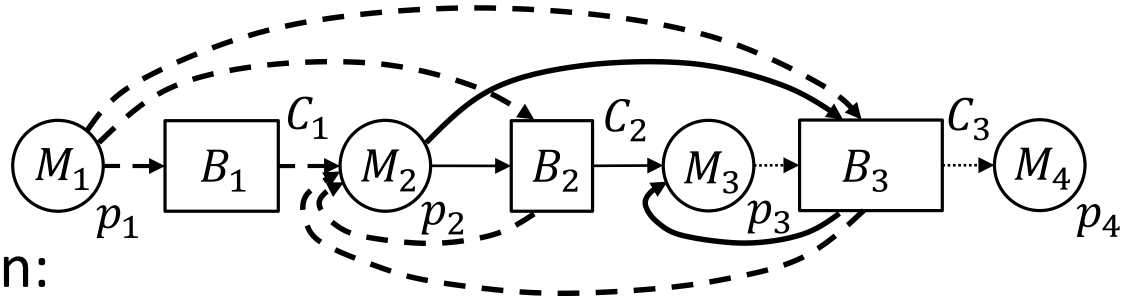
$$E_n = B_n \cup B_{n+1} \cup \cdots \cup B_{N-1} \cup M_{n+1}, n = 1, \dots, N-1$$

  - e_n = echelon WIP, $n = 1, \dots, N-1$

$$e_n \leq 1 + \sum_{m=n}^{N-1} C_m \equiv \text{ capacity of } E_n$$

EB policy

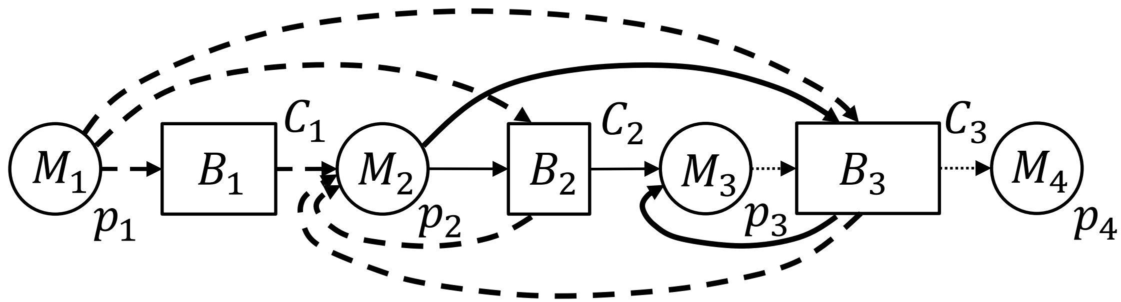
- Serial production line operated under EB policy



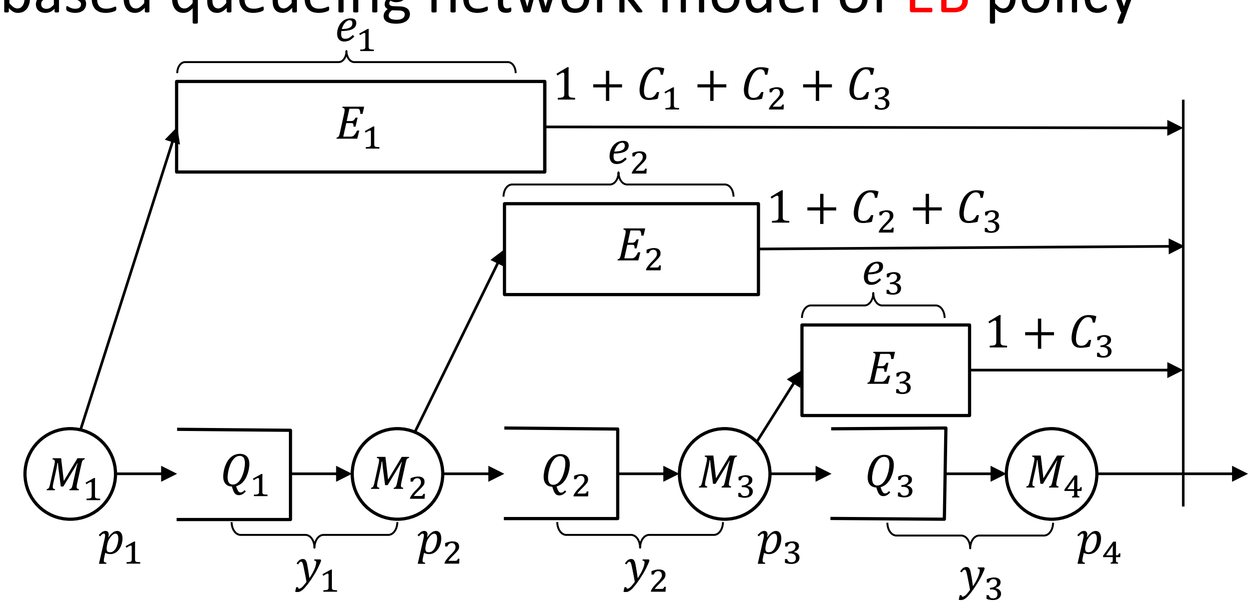
- Operation:
  - Machine M_n is allowed to store the parts that it produces in any of its downstream buffers $B_n, \ldots, B_{N-1}$ if the next machine M_{n+1} is occupied, $n = 1, \ldots, N-1$.
  - Machine M_n is blocked before service from processing a part if the number of parts that have been produced by it but have not yet departed from the last machine M_N is equal to $1 + \sum_{m=n}^{N-1} C_m$, i.e.,

$$e_n = 1 + \sum_{m=n}^{N-1} C_m$$

EB policy



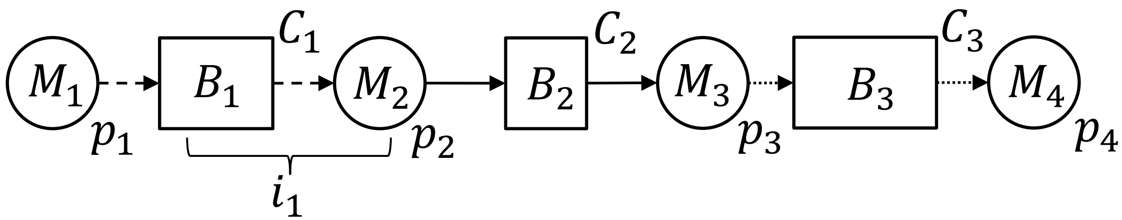
- Token-based queueing network model of EB policy



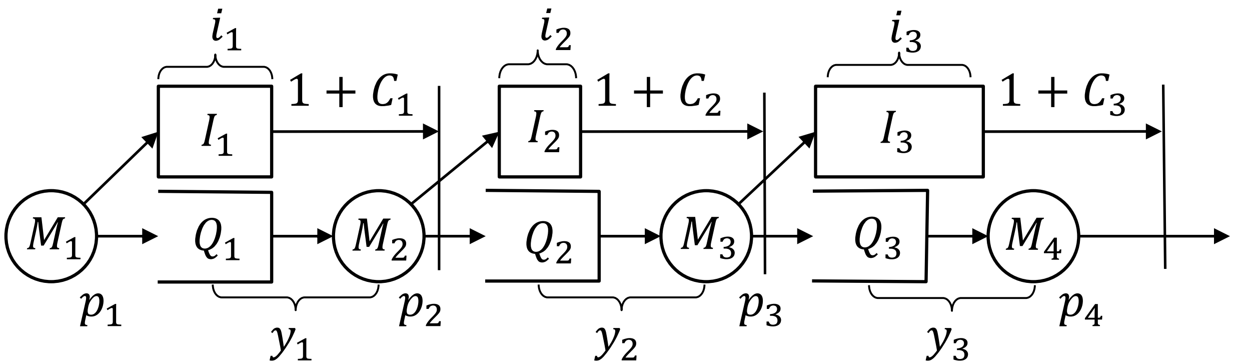
  - $y_n \equiv$ stage WIP, $n = 1, \ldots, N-1$

$$e_n = \sum_{m=n}^{N-1} y_m, n = 1, \ldots, N-1$$

Back to IB policy



- Token-based queueing network model of IB policy

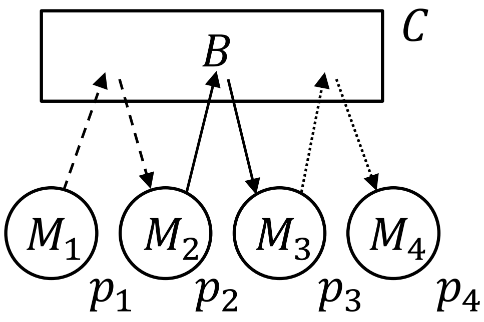


  - Q_n = stage-n queue, $n = 1, \ldots, N-1$
  - y_n = stage WIP, $n = 1, \ldots, N-1$

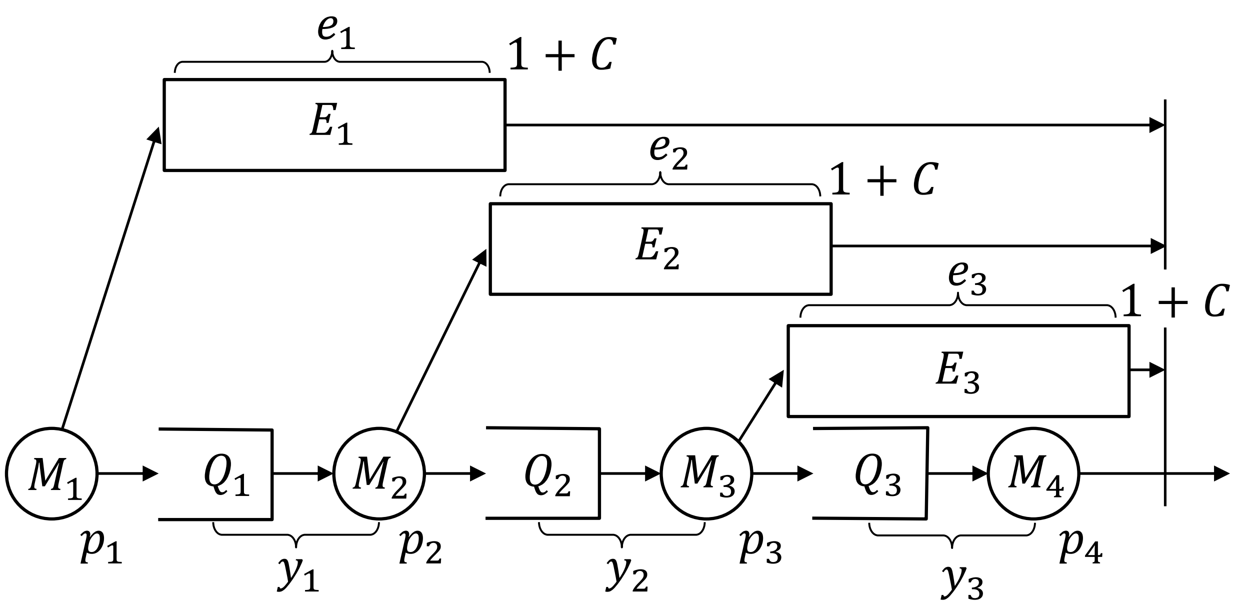
$$i_n = y_n, n = 1, \ldots, N-1$$

EB policy special case

- $C_n = 0, n = 1, \ldots, N-2$ and $C_{N-1} = C \geq 0$: CONWIP



- Token-based queueing network model of EB policy



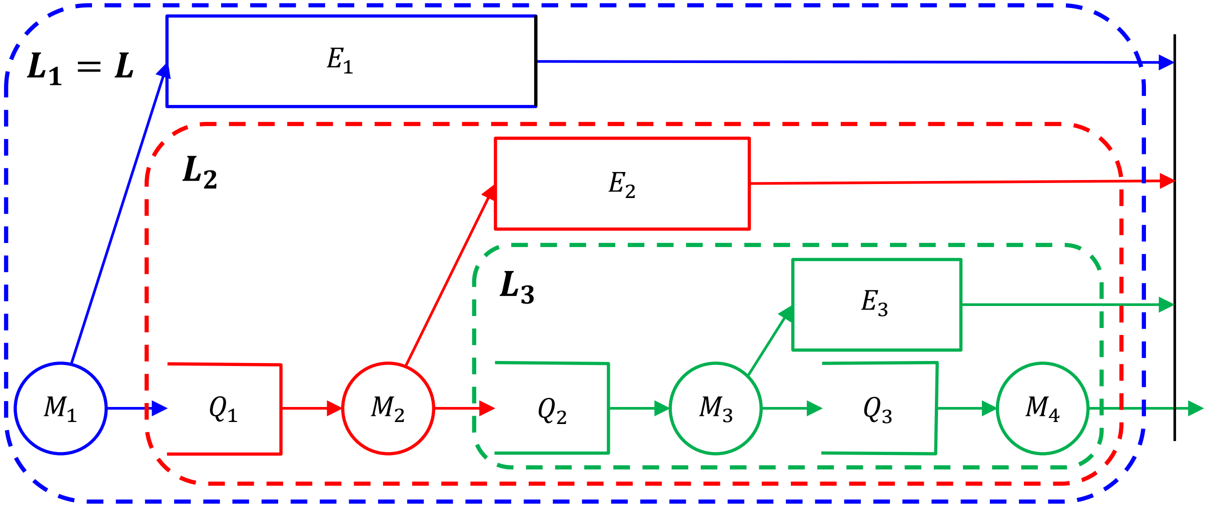
Advantages/disadvantages of EB

- Advantages of EB policy
  - Better utilization of buffer space
  - Global information
  - Fewer blockages and starvations of machines
  - Higher average throughput
- Disadvantages of EB policy
  - Higher average WIP
  - Higher transportation cost of parts to remote buffers
- To evaluate the advantages and disadvantages, need to:
  - Evaluate the performance of the EB policy
  - Optimize the parameters (buffer capacities) of the EB policy
  - Compare the optimal EB policy against the optimal IB and CONWIP policies

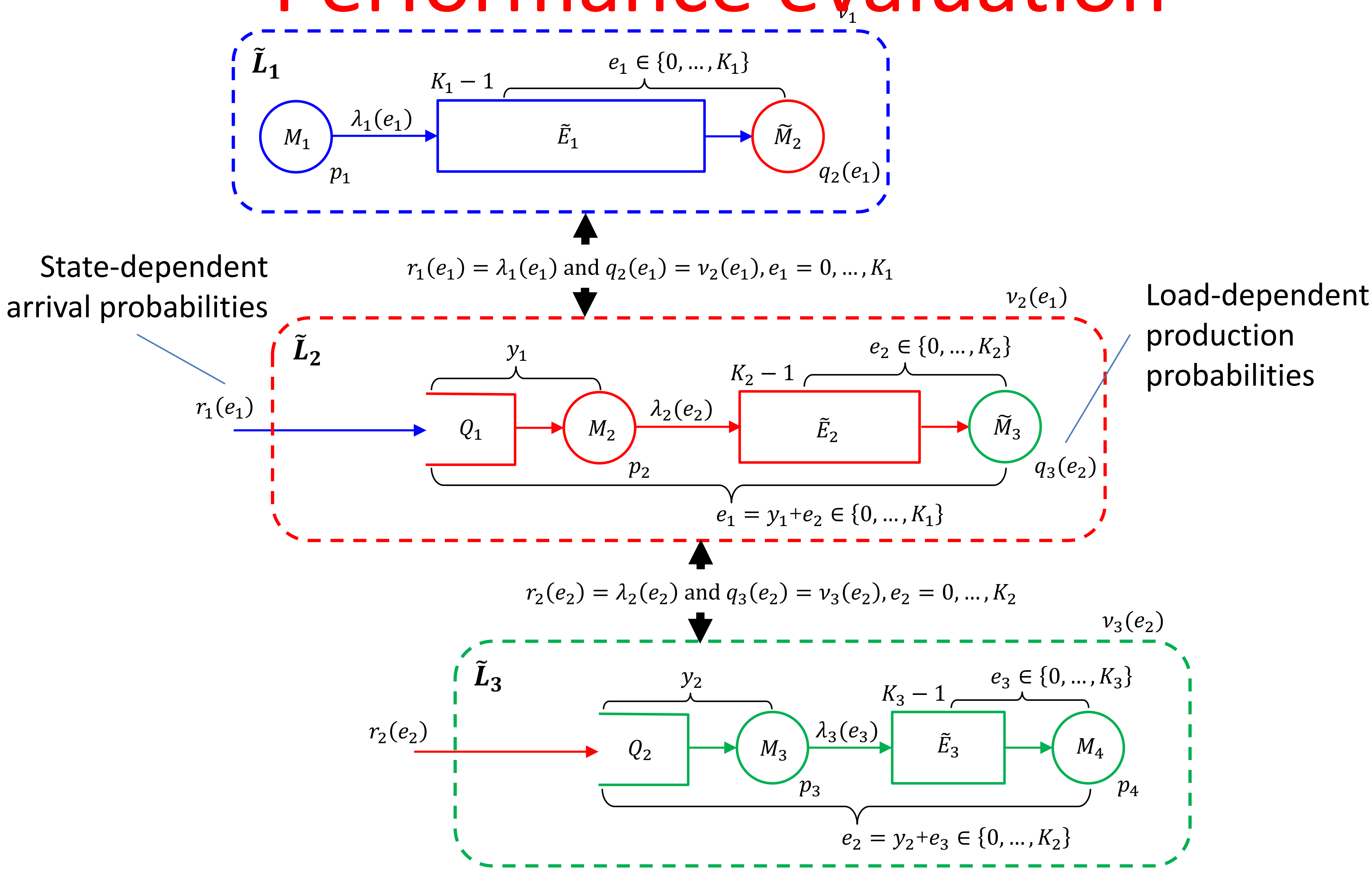
Performance evaluation

- Decomposition-based approximation method:
  1. Decompose the original queueing network of N machines and $N-1$ buffers into $N-1$ nested segments.
  2. Approximate each segment with a 2-machine subsystem than can be analyzed in isolation as a 2D MC.
  3. Determine the parameters of the 2-machine subsystems by relationships among the flows of parts in the original system.
- The approximation method is highly accurate and computationally efficient.

Performance evaluation



Performance evaluation



Buffer Allocation Problem (BAP)

Optimization problem (Shi and Gershwin 2009 adaptation):

Average net profit

Total buffer capacity

Overflow rate of stage WIP buffer Y_n

Average stage WIP

$$\max_{C_1,\ldots,C_{N-1}} P(C_1,\ldots,C_{N-1}) =$$

$$r\,\nu(C_1,\ldots,C_{N-1}) - \left(b\sum_{n=1}^{N-1} C_n + \sum_{n=1}^{N-1} h_n \bar{y}_n(C_1,\ldots,C_{N-1}) + t\sum_{n=1}^{N-1} \theta_n(C_1,\ldots,C_{N-1}) \right)$$

Average throughput

s.t. $\nu(C_1,\ldots,C_{N-1}) \geq \nu_{min}$

$C_n \geq 0, n = 1,\ldots,N-1$

θ_n = Prob [M_n produces a part AND $y_n \geq 1 + C_n$]

r = gross marginal profit (€ per part produced)

b = storage space cost (€ per storage slot per unit time)

h_n = stage WIP inventory holding cost (€ per part in y_n per unit time)

t = cost rate of transferring parts to remote buffers (€ per part transferred);

ν_{min} = minimum required average throughput

BAP: Solution methodology

Solution methodology (Shi and Gershwin 2009 adaptation):

1. Solve unconstrained problem (using "gradient" search: step by step increments)

If $v(C_1^*, \dots, C_{N-1}^*) \geq v_{min}$ then GOTO EXIT

2. (Else) set $r = r + \Delta r$ and resolve unconstrained problem (Lagrange multiplier method)

If $v(C_1^*, \dots, C_{N-1}^*) \geq v_{min}$ then GOTO EXIT

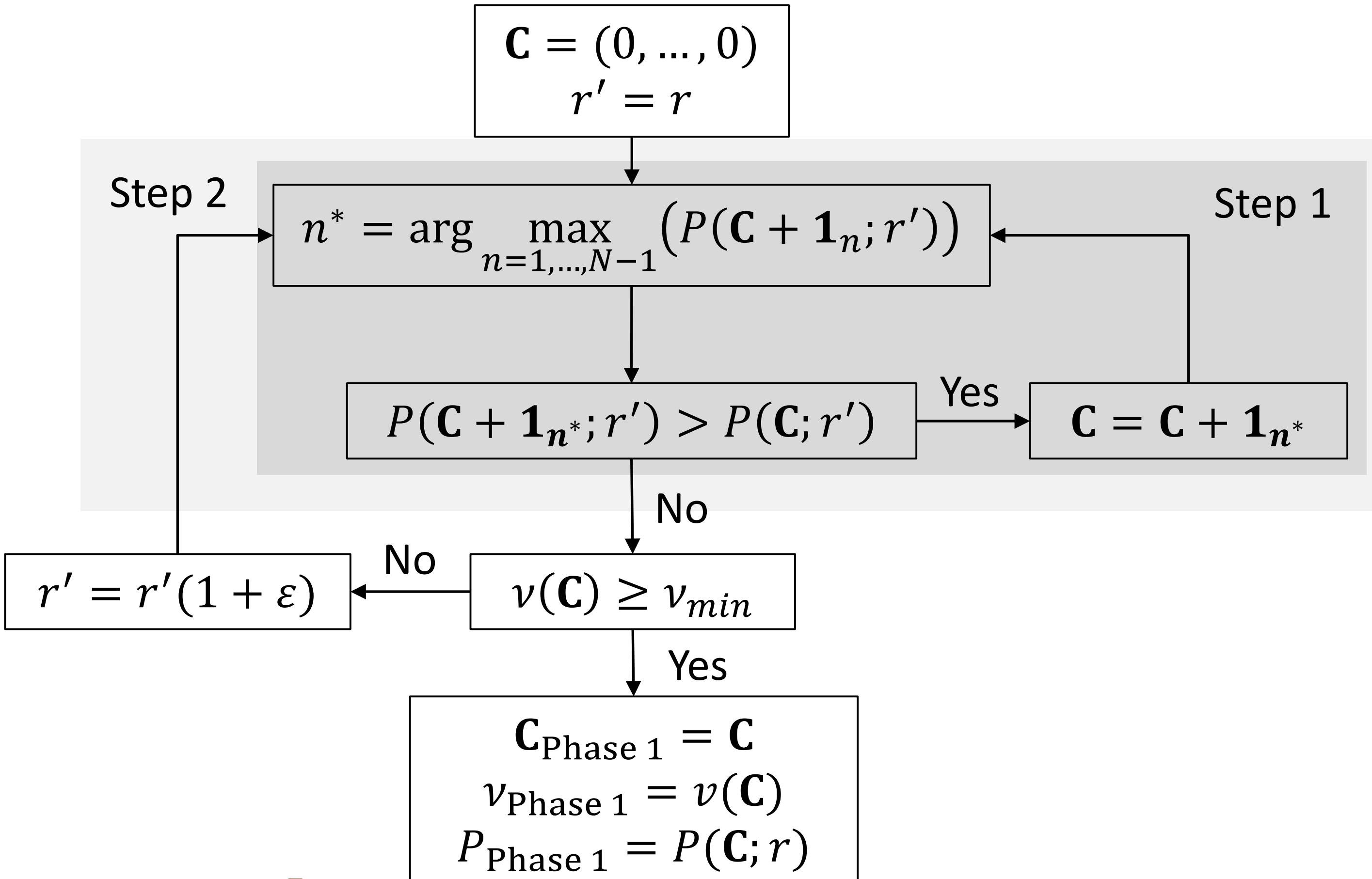
Else GOTO step 2

EXIT: Compute final maximum average profit using original value of r

(Phase 2: perform a local search to improve solution)

BAP: Solution methodology

- Solution methodology (Shi and Gershwin 2009 adaptation):



Numerical results: Input parameters

- Experimental design:
  - To systematically compare the IB, EB, and CONWIP policies, we optimized each policy for an $N = 8$ machine line and several scenarios for the
    - Production rates $p_n, n = 1, \dots., N$
    - Parameters $r, b, h_n, n = 1, \dots, N-1, v_{min}$
    - Parameter t

Numerical results: Input parameters

- Production rate scenarios

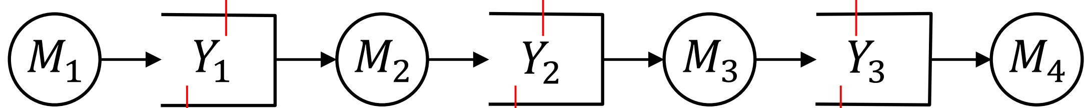
#	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8
L_1	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
L_2	0.6	**0.5**	0.6	0.6	0.6	0.6	0.6	0.6
L_3	0.6	0.6	0.6	0.6	**0.5**	0.6	0.6	0.6
L_4	0.6	0.6	0.6	0.6	0.6	0.6	**0.5**	0.6
L_5	0.53	0.55	0.57	0.59	0.61	0.63	0.65	0.67
L_6	0.67	0.65	0.63	0.61	0.59	0.57	0.55	0.53

Numerical results: Input parameters

Scenarios for parameters $r, b, h_n, n = 1, \dots, N-1, v_{min}$: Auxiliary input parameters

$I_c = 0.5$ $\quad c_0 = 100$ $\quad c_1 = 150$ $\quad c_2 = 200$ $\quad c_3 = 250$ $\quad c_4 = 300$

$I_r = 0.1$



$r = I_r c_N = 30$

$I_h = 0.001$ $\quad h_1 = 0.15$ $\quad h_2 = 0.2$ $\quad h_3 = 0.25$

$I_b = 2$ $\quad b = I_b h_1 = 0.3$ $\quad I_v = 0.8$ $\quad v_{min} = I_v p_{min} = 0.8\, p_{min}$

c_0 = raw-material cost (€ per raw part)

I_c = value-added rate per production stage as a percentage of c_0

c_n = total cost per part produced by M_n (€ per part) $\quad c_n = c_{n-1} + I_c c_0 = c_0(1 + nI_c)$

I_h = interest rate (€ per € invested per unit time) $\quad h_n = I_h c_n$

I_r = gross profit margin $\quad r = I_r c_N$

I_b = buffer space cost multiplier w.r.t. h_1 $\quad b = I_b h_1$

I_v = minimum required production line efficiency $\quad v_{min} = I_v p_{min}$

Numerical results: Input parameters

Scenarios for parameters $r, b, h_n, n = 1, \dots, N-1$, based on auxiliary input parameters

	Auxiliary parameters						Optimization parameters								
#	c_0	I_c	I_h	I_r	I_b	I_v	r	b	h_1	h_2	h_3	h_4	h_5	h_6	h_7
O_1	100	0.5	0.001	0.10	2	0.8	50	0.30	0.15	0.2	0.25	0.3	0.35	0.4	0.45
O_2	100	**0.0**	0.001	0.10	2	0.8	10	0.20	0.10	0.1	0.10	0.1	0.10	0.1	0.10
O_3	100	**1.0**	0.001	0.10	2	0.8	90	0.40	0.20	0.3	0.40	0.5	0.60	0.7	0.80
O_4	100	**5.0**	0.001	0.10	2	0.8	410	1.20	0.60	1.1	1.60	2.1	2.60	3.1	3.60
O_5	100	**10.0**	0.001	0.10	2	0.8	810	2.20	1.10	2.1	3.10	4.1	5.10	6.1	7.10
O_6	100	0.5	**0.002**	0.10	2	0.8	50	0.60	0.30	0.4	0.50	0.6	0.70	0.8	0.90
O_7	100	0.5	0.001	**0.20**	2	0.8	100	0.30	0.15	0.2	0.25	0.3	0.35	0.4	0.45
O_8	100	0.5	0.001	**0.05**	2	0.8	25	0.30	0.15	0.2	0.25	0.3	0.35	0.4	0.45
O_9	100	0.5	0.001	0.10	**0**	0.8	50	0.00	0.15	0.2	0.25	0.3	0.35	0.4	0.45
O_{10}	100	0.5	0.001	0.10	**1**	0.8	50	0.15	0.15	0.2	0.25	0.3	0.35	0.4	0.45
O_{11}	100	0.5	0.001	0.10	2	**0.0**	50	0.30	0.15	0.2	0.25	0.3	0.35	0.4	0.45

Numerical results: Input parameters

- Scenarios for transfer cost rate t

I_t = transfer cost rate as multiple of raw material cost c_0 $t = I_t c_0$

$c_0 = 100$ $I_t = 0.01$ $t = 1$

#	I_t		t
T_1	0.00		0
T_2	0.01		1
T_3	0.02		2

Numerical results: Experiments

- We run all combinations of scenarios $L_1 - L_6, T_1 - T_3, O_1 - O_{11}$
- Total number of instances: $6 \times 3 \times 11 = 198$
- In each instance, we optimized the installation buffer capacities for:
  – Installation buffer (IB) policy $(C_1, \dots, C_7)$
  – Echelon buffer (EB) policy $(C_1, \dots, C_7)$
  – CONWIP policy $(C \equiv C_7;\ C_1 = \cdots = C_6 = 0)$

Numerical results: Optimal values

- Results for scenarios $L_1, T_1, O_1 - O_{11}$ (balanced line, $t = 0$)

		IB Policy			EB Policy			CONWIP Policy			$\%\Delta P^*$	
	v_{min}	$[C_1^*\text{-}C_7^*]$	v^*	P^*	$[C_1^*\text{-}C_7^*]$	v^*	P^*	C^*	v^*	P^*	E-I	E-C
O_1	0.480	[2,3,4,3,4,3,3]	0.480	13.465	[0,0,0,1,0,5,8]	0.483	15.991	14	0.483	15.991	18.757	0.0009
O_2	0.480	[2,3,4,3,4,3,3]	0.480	-0.926	[0,0,0,0,0,5,9]	0.483	0.713	14	0.483	0.713	176.979	0.0011
O_3	0.480	[2,3,4,3,4,3,3]	0.480	27.857	[0,0,0,1,0,5,8]	0.483	31.268	14	0.483	31.268	12.247	0.0009
O_4	0.480	[2,3,4,3,4,3,3]	0.480	142.989	[0,0,0,1,0,6,7]	0.483	153.490	14	0.483	153.488	7.344	0.0011
O_5	0.480	[3,3,3,4,3,4,3]	0.485	286.147	[0,0,1,0,2,4,8]	0.490	306.436	15	0.490	306.431	7.090	0.0015
O_6	0.480	[2,3,4,3,4,3,3]	0.480	2.915	[0,0,1,2,1,3,7]	0.483	7.854	14	0.483	7.853	169.458	0.0098
O_7	0.480	[2,3,4,3,4,3,3]	0.480	37.481	[0,0,0,0,2,5,10]	0.502	40.400	17	0.502	40.400	7.789	0.0004
O_8	0.480	[2,3,4,3,4,3,3]	0.480	1.457	[0,0,1,2,1,3,7]	0.483	3.927	14	0.483	3.927	169.458	0.0098
O_9	0.480	[2,5,5,5,5,5,5]	0.502	20.460	[0,0,0,1,1,6,9]	0.502	20.388	17	0.502	20.388	-0.352	0.0010
O_{10}	0.480	[2,3,4,3,4,3,3]	0.480	16.765	[0,0,0,1,0,5,8]	0.483	18.091	14	0.483	18.091	7.907	0.0008
O_{11}	0.000	[1,2,2,2,2,2,2]	0.421	14.515	[0,0,0,0,1,3,7]	0.454	16.234	11	0.454	16.234	11.843	0.0004

Numerical results: Optimal values

- Results for scenarios $L_1, T_2, O_1 - O_{11}$ (balanced line, $t = 0.01$)

		IB Policy			EB Policy			CONWIP Policy			$\%\Delta P^*$	
	v_{min}	$[C_1^*\text{-}C_7^*]$	v^*	P^*	$[C_1^*\text{-}C_7^*]$	v^*	P^*	C^*	v^*	P^*	E-I	E-C
O_1	0.480	[2,3,4,3,4,3,3]	0.480	13.465	[0,1,1,2,2,3,5]	0.482	15.504	14	0.483	15.067	15.142	2.8999
O_2	0.480	[2,3,4,3,4,3,3]	0.480	-0.926	[0,1,2,2,2,3,4]	0.481	0.268	14	0.483	-0.210	128.876	227.1177
O_3	0.480	[2,3,4,3,4,3,3]	0.480	27.857	[0,1,1,2,2,3,5]	0.482	30.767	14	0.483	30.345	10.449	1.3935
O_4	0.480	[2,3,4,3,4,3,3]	0.480	142.989	[0,1,1,1,2,3,6]	0.482	152.942	14	0.483	152.564	6.961	0.2477
O_5	0.480	[3,3,3,4,3,4,3]	0.485	286.147	[0,0,1,2,2,3,7]	0.490	305.865	15	0.490	305.478	6.891	0.1265
O_6	0.480	[2,3,4,3,4,3,3]	0.480	2.915	[1,1,1,2,2,2,5]	0.480	7.391	14	0.483	6.930	153.565	6.6543
O_7	0.480	[2,3,4,3,4,3,3]	0.480	37.481	[0,1,2,2,2,4,6]	0.502	39.850	16	0.496	39.401	6.322	1.1394
O_8	0.480	[2,3,4,3,4,3,3]	0.480	1.457	[1,1,1,2,2,2,5]	0.480	3.482	14	0.483	3.003	138.934	15.9541
O_9	0.480	[2,5,5,5,5,5,5]	0.502	20.460	[1,1,2,2,2,3,6]	0.501	19.880	17	0.502	19.384	-2.832	2.5595
O_{10}	0.480	[2,3,4,3,4,3,3]	0.480	16.765	[0,1,1,2,2,3,5]	0.482	17.604	14	0.483	17.167	5.003	2.5451
O_{11}	0.000	[1,2,2,2,2,2,2]	0.421	14.515	[0,1,1,1,2,2,4]	0.453	15.793	11	0.454	15.423	8.802	2.3997

Numerical results: Optimal values

- Results for scenarios $L_1, T_3, O_1 - O_{11}$ (balanced line, $t = 0.02$)

		IB Policy			EB Policy			CONWIP Policy			$\%\Delta P^*$	
	v_{min}	$[C_1^*\text{-}C_7^*]$	v^*	P^*	$[C_1^*\text{-}C_7^*]$	v^*	P^*	C^*	v^*	P^*	E-I	E-C
O_1	0.480	[2,3,4,3,4,3,3]	0.480	13.465	[1,1,1,2,2,2,5]	0.480	15.064	14	0.483	14.143	11.874	6.5083
O_2	0.480	[2,3,4,3,4,3,3]	0.480	-0.926	[0,1,2,2,2,3,4]	0.481	-0.159	14	0.483	-1.134	82.846	85.9868
O_3	0.480	[2,3,4,3,4,3,3]	0.480	27.857	[0,1,1,2,2,3,5]	0.482	30.301	14	0.483	29.421	8.775	2.9910
O_4	0.480	[2,3,4,3,4,3,3]	0.480	142.989	[0,1,1,1,2,3,6]	0.482	152.437	14	0.483	151.641	6.607	0.5248
O_5	0.480	[3,3,3,4,3,4,3]	0.485	286.147	[0,1,1,1,2,3,7]	0.490	305.323	15	0.490	304.525	6.702	0.2619
O_6	0.480	[2,3,4,3,4,3,3]	0.480	2.915	[1,1,1,2,2,2,5]	0.480	6.964	14	0.483	6.006	138.934	15.9541
O_7	0.480	[2,3,4,3,4,3,3]	0.480	37.481	[1,1,1,2,3,3,6]	0.501	39.351	16	0.496	38.422	4.991	2.4192
O_8	0.480	[2,3,4,3,4,3,3]	0.480	1.457	[1,1,1,2,2,2,5]	0.480	3.056	14	0.483	2.079	109.670	46.9451
O_9	0.480	[2,5,5,5,5,5,5]	0.502	20.460	[1,1,2,2,3,3,5]	0.500	19.436	16	0.496	18.400	-5.005	5.6281
O_{10}	0.480	[2,3,4,3,4,3,3]	0.480	16.765	[1,1,1,2,2,2,5]	0.480	17.164	14	0.483	16.243	2.379	5.6669
O_{11}	0.000	[1,2,2,2,2,2,2]	0.421	14.515	[0,1,1,1,2,2,4]	0.453	15.387	10	0.441	14.631	6.004	5.1630

Numerical results: Optimal values

- Results for scenarios $L_2, T_2, O_1 - O_{11}$ (bottleneck upstream, $t = 0.01$)

		IB Policy			EB Policy			CONWIP Policy			$\%\Delta P^*$	
	v_{min}	$[C_1^*\text{-}C_7^*]$	v^*	P^*	$[C_1^*\text{-}C_7^*]$	v^*	P^*	C^*	v^*	P^*	E-I	E-C
O_1	0.400	[2,3,2,2,2,2,2]	0.424	13.829	[0,1,1,1,2,2,4]	0.433	14.951	10	0.422	14.584	8.115	2.5177
O_2	0.400	[2,2,2,2,2,2,1]	0.409	0.513	[0,1,1,1,2,2,2]	0.406	1.049	9	0.409	0.694	104.365	51.0399
O_3	0.400	[2,3,2,2,2,2,2]	0.424	27.434	[0,1,1,1,2,3,4]	0.443	29.329	12	0.444	28.943	6.909	1.3347
O_4	0.400	[2,3,3,3,2,2,2]	0.435	136.946	[0,1,1,1,2,3,6]	0.459	145.855	14	0.459	145.452	6.505	0.2770
O_5	0.400	[3,4,3,3,3,3,2]	0.457	276.487	[0,1,1,1,2,3,6]	0.459	291.752	14	0.459	291.358	5.521	0.1354
O_6	0.400	[2,2,2,2,2,2,1]	0.409	7.097	[0,1,1,1,1,2,3]	0.408	9.573	9	0.409	9.259	34.891	3.3905
O_7	0.400	[3,4,3,3,3,3,2]	0.457	35.809	[0,1,2,2,2,3,5]	0.465	37.605	15	0.465	37.150	5.014	1.2237
O_8	0.400	[2,2,2,2,2,2,1]	0.409	3.548	[0,1,1,1,1,2,3]	0.408	4.608	9	0.409	4.275	29.868	7.7805
O_9	0.400	[4,11,5,5,4,4,4]	0.483	19.821	[0,2,2,2,3,3,5]	0.474	18.921	16	0.470	18.410	-4.542	2.7726
O_{10}	0.400	[2,3,3,2,3,2,2]	0.434	16.146	[0,1,1,2,2,3,4]	0.452	16.718	12	0.444	16.291	3.541	2.6241
O_{11}	0.000	[1,2,2,2,1,2,1]	0.384	13.552	[0,1,1,1,2,2,4]	0.433	14.951	10	0.422	14.584	10.328	2.5177

Numerical results: Optimal values

- Results for scenarios $L_4, T_2, O_1 - O_{11}$ (bottleneck downstream, $t = 0.01$)

		IB Policy			EB Policy			CONWIP Policy			$\%\Delta P^*$	
	v_{min}	$[C_1^*\text{-}C_7^*]$	v^*	P^*	$[C_1^*\text{-}C_7^*]$	v^*	P^*	C^*	v^*	P^*	E-I	E-C
O_1	0.400	[1,2,2,2,2,3,2]	0.417	13.557	[0,1,1,1,1,2,4]	0.421	14.650	10	0.422	14.337	8.058	2.1796
O_2	0.400	[1,1,3,2,2,2,2]	0.401	0.500	[0,1,1,1,1,2,3]	0.406	1.037	9	0.409	0.694	107.437	49.3610
O_3	0.400	[1,2,2,2,2,3,2]	0.417	26.758	[0,1,1,1,1,2,5]	0.433	28.599	11	0.434	28.284	6.882	1.1153
O_4	0.400	[2,2,2,2,3,3,3]	0.436	133.684	[0,1,1,1,1,2,7]	0.452	141.276	12	0.444	140.960	5.679	0.2242
O_5	0.400	[2,2,2,2,3,3,3]	0.436	267.597	[0,0,1,1,2,2,7]	0.452	282.420	13	0.452	282.108	5.539	0.1107
O_6	0.400	[1,2,2,1,2,3,2]	0.401	6.557	[0,0,1,1,1,2,4]	0.408	9.146	9	0.409	8.858	39.487	3.2533
O_7	0.400	[2,2,2,3,3,4,3]	0.447	35.154	[0,1,1,1,2,3,6]	0.459	36.953	14	0.459	36.588	5.118	0.9978
O_8	0.400	[1,2,2,1,2,3,2]	0.401	3.278	[0,1,1,1,1,2,3]	0.406	4.399	9	0.409	4.075	34.180	7.9516
O_9	0.400	[1,5,4,4,4,4,9]	0.461	18.852	[1,1,1,1,2,3,5]	0.457	18.239	14	0.459	17.833	-3.251	2.2752
O_{10}	0.400	[1,2,2,2,2,3,3]	0.420	15.716	[0,1,1,1,1,3,5]	0.443	16.294	12	0.444	15.930	3.678	2.2836
O_{11}	0.000	[1,1,2,2,2,2,2]	0.394	13.516	[0,1,1,1,1,2,4]	0.421	14.650	10	0.422	14.337	8.389	2.1796

Conclusions

- We performed an extended numerical study comparing the optimal IB, EB and CONWIP policies
- EB significantly outperforms IB ($t = 0$)
- EB barely outperforms CONWIP ($t = 0$)
- As $t \nearrow$, dominance of EB over IB $\searrow$
- As $t \nearrow$, dominance of EB over CONWIP $\nearrow$
- Total buffer capacity: CONWIP $= (<)$ EB << IB
- Numerically verified that the objective function of the EB policy is concave

Direction for future research

- Consider more complex machine models than the Bernoulli model

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Thank you for your attention.

Any questions?