The Impact of Orbit Dependent Return Rate on the Control Policies of a Hybrid Production System

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Production Control and Return Flows

Objective: Production control under uncertain return flows.
• Returns may affect inventory levels.
• Return flow estimates are required.
• Returns and supplied demand are dependent through products in use
Literature assumes demand and return flows to be independent in most cases.

Demand generates new users when the sales is not for replacement.

The more new products are sold to new users the higher the number of products in use (in orbit).

The size of the orbit determines the return rate.

When some of the products in orbit are disposed off rather than being returned an unobservable decrease occurs in the products in use.

The return rate only gives a partial information about this fact.
Literature on Optimal Control Models

How would the information about orbit affect the production system control?

- Toktay et. al (2000)
  - CLSC model using QN with \( /M/\infty \) queue representing users

- Flapper et. al (2012)
  - Optimal production control with advanced return info.

- Zerhouni et. al (2013)
  - Comparison of dependent vs. independent returns under base stock control.
  - Continuous review base stock optimal for independent returns
  - Instantaneous usage rate assumption for dependent returns
Objectives of the Study

- How does the production decision change under return flows dependent on the size of the orbit?
- What is the value added of using return flows in manufacturing?
- What is the cost of ignoring/inaccurately estimating the information of orbit size?
The Model

\[ \lambda : \text{Demand rate} \]
\[ \mu : \text{Usage rate} \]
\[ \gamma : \text{Disposal rate} \]
\[ \theta : \text{Production rate} \]
\[ p = \frac{\mu}{(\gamma + \mu)} \]
\[ x_0 : \text{Orbit size} \]
\[ x_I : \text{Inv. level} \]
Assumptions

• Each product in orbit can be disposed of with a positive probability.
• No need for remanufacturing (or instant remanufacturing to as good as new).
• All returns are accepted.
• The system is continuously observed.
• $x_0$ is fully observable.
Value Function

\[ v(x) + g = \Lambda_2^{-1} \left[ hx_I^+ + \mu x_O v(x_O - 1, x_I + 1) \right. \]

\[ + \gamma x_O v(x_O - 1, x_I) \]

\[ \left. + \lambda \begin{cases} v(x_O + 1, x_I - 1), & x_I > 0 \\ v(x_O, x_I) + c_L, & x_I = 0 \end{cases} \right\} + \min \theta \{ v(x_O, x_I + 1), v(x_O, x_I) \} \right] \]

\[ \Lambda_1 = (\mu + \gamma) x_O^{\text{max}} + \lambda + \theta \]

\( h \): holding cost

\( c_L \): lost sales cost
Optimal Production Control w. Returns

\[ \theta = 1, \lambda = 1.2, \gamma = 0.002, \mu = 0.018, (p = 0.9) \]

\[ c_L/h = 10, S_{max} = 3, \]

\[ c_L/h = 50, S_{max} = 9, \]
Zerhouni et al. (2013) show for a state independent stationary return flow the optimal control policy is base stock.

State independent return rate estimates are needed.

An estimate can be given as $\hat{\mu} = \alpha \cdot \bar{\mu}$, $\alpha \in [0, \infty)$
where $\bar{\mu} = E[x_0] \cdot \mu$.

Thus the average orbit size is required.
The LP equivalent of the value function calculates the stationary distribution $P(x_I, x_0)$ (Büyükdağlı and Fadıloğlu, 2016).

$$\min Z = \sum_{x \in S} \left( c_{x,0} \cdot y_{x,0} + c_{x,1} \cdot y_{x,1} \right)$$

s.t.

$$\sum_{x \in S} (y_{x,0} + y_{x,1}) = 1$$

$$y_{x,0} + y_{x,1} - \Lambda^{-1} \left[ \theta \left( y_{x,0} + y_{x-e_I,1} \right) + \lambda \left( y_{x-e_0+e_I,0} + y_{x-e_0+e_I,1} \right) 
+ x_0 \mu \left( y_{x+e_0-e_I,0} + y_{x+e_0-e_I,1} \right) + x_0 \gamma \left( y_{x-e_I,0} + y_{x-e_I,1} \right)
+ \left( M - x_0 \right) \left( \gamma + \mu \right) \left( y_{x,0} + y_{x,1} \right) \right] = 0$$

$\forall y_{x,0} \geq 0, y_{x,1} \geq 0$

$\forall x \in S$

$x = (x_0, x_I), e_0 = (1,0), e_I = (0,1)$

$c_{x,d} = h \cdot x_I + (1-x_I)^+ \cdot \lambda \cdot c_L, d = \{0,1\}$
The Procedure

\[ \hat{\mu} = \alpha \cdot E[x_0] \cdot \mu \text{ from LP} \]

\[ S_{\hat{\mu}} \text{ from } v(x_I) \]

\[ S_{\hat{\mu}} \text{ to } v(x) = v_{S_{\hat{\mu}}}(x) \]

Compare \( v(x) \) & \( v_{S_{\hat{\mu}}}(x) \)

\( v(x_I) \) is the value function for optimizing the FGI without using the orbit information. \( S_{\hat{\mu}} \) is the optimal base-stock level for \( v(x_I) \).
Decision Matrix from Value Iteration

\[ \mu = \gamma = 0.006 \]

\[ \mu = \gamma = 0.007 \]

\[ \mu = \gamma = 0.008 \]

\[ \mu = \gamma = 0.01 \]

\[ \lambda = 1, \theta = 1, \mu / \gamma = 1, c_L / h = 50 \]
Stationary Distribution from LP

\[ \mu = \gamma = 0.006 \]

\[ \mu = \gamma = 0.007 \]

\[ \mu = \gamma = 0.008 \]

\[ \mu = \gamma = 0.01 \]
Marginal Probability of Inventory Level

\[ \mu = \gamma = 0.006 \]

\[ \mu = \gamma = 0.007 \]

\[ \mu = \gamma = 0.008 \]

\[ \mu = \gamma = 0.01 \]
Marginal Probability of Orbit Size

\[ \mu = \gamma = 0.006 \]

\[ \mu = \gamma = 0.007 \]

\[ \mu = \gamma = 0.008 \]

\[ \mu = \gamma = 0.01 \]
Pure vs. Hybrid Production

For $\alpha = 0$ we can observe the added value of using returns.
If no returns are accepted $S_{\text{max}}$ is the optimal control policy.

$$\theta = 1, \lambda = \{0.8, 1, 1.2\}, \ p = \mu/(\mu + \gamma) = \{0.1, 0.3, 0.5, 0.7, 0.9\},$$

$$\mu = \{0.002, \ldots, 0.018\}, c_L/h = \{10, 50\}, h = 1$$
Pure vs. Hybrid Production

• The impact of $p$ and $\lambda/\theta$
• As $\lambda/\theta$ increases the impact of returns increase since the orbit expands. For $p=0.9$ and $\lambda = 1.2, \bar{\mu} = 1.047$

![Graph showing error probabilities for different demand rates and return probabilities.](image)
$\hat{\mu} = \bar{\mu} \cdot \{0.5, 0.8, 1, 1.2,\}$
## Remanufacturing vs Manufacturing Cost

Control levels for $\bar{\mu}$ model under the impact of production costs.

$\theta = 1$, $c_L/h = 50$, $c_M = 5$, $c_R = 2$

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
p & \lambda = 0.8 & \lambda = 0.8 & \lambda = 0.8 \\
\hline
 & S \text{ with } C_{R-C_M} & S & S \text{ with } C_{R-C_M} & S & S \text{ with } C_{R-C_M} \\
\hline
0.1 & 4 & 5 & 6 & 7 & 10 & 10 \\
\hline
0.3 & 3 & 4 & 5 & 5 & 7 & 7 \\
\hline
0.5 & 3 & 3 & 4 & 4 & 5 & 5 \\
\hline
0.7 & 2 & 2 & 3 & 3 & 3 & 4 \\
\hline
0.9 & 1 & 1 & 1 & 1 & 2 & 2 \\
\hline
\end{array}
\]
Conclusions

• A testbed for measuring the impact of the number of products in use (orbit) on the production control of hybrid production systems is introduced.

• The added value of using returns for supplying the demand is clearly displayed.

• Results indicate that the base stock policy is heavily dependent on the orbit size and return probabilities.

• The difference between complete information and no information present the value of the partial observation effort.
Future Work

Model structure extensions:
• Return admission control
• Remanufacturing lead time
• Demand differentiation
• The feedback relation between the orbit and the demand rate.

Testbed extensions:
• Automated LP model
• LP model for upto S levels
• Measure the effectiveness of proposed methods so far
Thank You

References:
Zerhouni et. al (2013), Influence Of Dependency Between Demands And Returns In A Reverse Logistics System, IJPE 143.
Distributions of the given example.

\[ \theta = 1, \lambda = 1.2, \gamma = 0.002, \mu = 0.018, (p = 0.9), c_L/h = 50 \]

\[ E[x_I] = 6,195 \quad \bar{\mu} = 1,047, \quad E[x_0] = 58 \]

\[ P(x_I, x_0) \]