Approaches for machine selection and buffer allocation in stochastic flow lines

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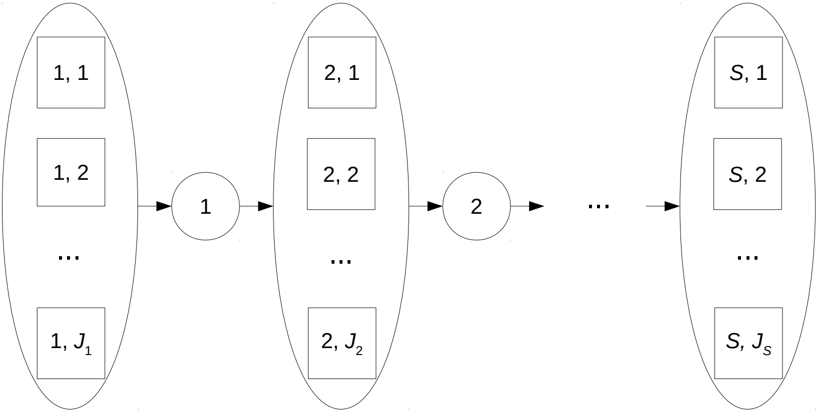
1. Problem and motivation
2. Conceptual optimization model
3. Brute force MIP model
4. LocalSolver plus flow line decomposition
5. Elements of a Branch & Bound approach
6. No numerical results, but ...

Stochastic flow lines with alternative machines



Problem

- Serial production process
- Single product with target production rate $PR^{\min}$
- Decision I: Selection of one the alternative machines $j = 1, ..., J_s$ with stochastic processing times $T_{s,j}$ for each station s
- Decision II: Capacity b_k of the buffer behind station $s = 1, ..., S - 1$

- Objective: Minimize required capital budget for machines and buffers

Conceptual model

$$\text{Min } = \sum_{s=1}^{S}\sum_{j=1}^{J_s} cr^M_{s,j}\cdot v_{s,j} + \sum_{s=1}^{S-1} cr^B_s \cdot x_s$$

$$\sum_{j=1}^{J_s} v_{s,j} = 1, \qquad s = 1, ..., S$$

$$PR(\underline{v}, \underline{x}) \geq PR^{\min}$$

$$v_{s,j} \in \{0,1\}, \qquad s = 1, ..., S; j = 1, ..., J_s$$

$$x_s \in \{0,1,2,3,.....\}, \qquad s = 1, ..., S-1$$

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Difficulties: $PR(\underline{v}, \underline{x})$ non-linear, no closed-form expression, integrality constraints on decision variables

Solving the model

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- Performance evaluation methodology
- Optimization methodology

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Approaches:

- Simulation optimization in an LP
- LocalSolver plus decomposition
- Branch & Bound plus decomposition

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- Very general and flexible, very time-consuming
- Limited usefulness, computation of reference values

$$\text{Min } = \sum_{s=1}^{S}\sum_{j=1}^{J_s} cr_{s,j}^{M} \cdot V_{s,j} + \sum_{s=1}^{S-1} cr_s^B \cdot X_s \tag{1}$$

$$\sum_{j=1}^{J_s} V_{s,j} = 1, \qquad \forall s \tag{2}$$

$$XS_{s,w} + \sum_{j=1}^{J_s} d_{s,j,w} \cdot V_{s,j} \leq XF_{s,w}, \qquad \forall s, \forall w \tag{3}$$

$$XF_{s,w} \leq XS_{s+1,w}, \qquad \forall s \leq S-1, \forall w \tag{4}$$

$$XF_{s,w} \leq XS_{s,w+1}, \qquad \forall s, \forall w \leq W-1 \tag{5}$$

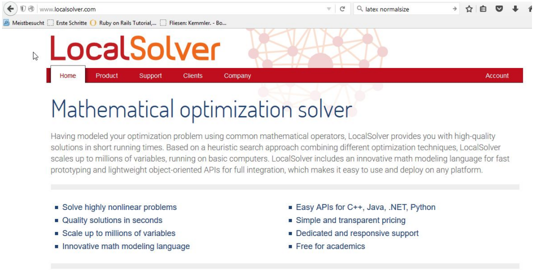
$$XF_{S,W} - XF_{S,W_0} \leq \frac{W - W_0}{PR^{\min}} \tag{6}$$

$$XS_{s+1,w} - XF_{s,w+b} \leq M \cdot (1 - Y_{s,b}), \qquad \forall s \leq S-1, \forall b, \forall w \leq W-b \tag{7}$$

$$\sum_{b=0}^{B_s} Y_{s,b} = 1, \qquad \forall s \leq S-1 \tag{8}$$

$$X_s = \sum_{b=0}^{B_s} b \cdot Y_{s,b}, \qquad \forall s \leq S-1 \tag{9}$$

LocalSolver-Approach



- Commercial software, academic licenses
- Heuristic search algorithms
- Combinatorial problems, discrete decision variables
- Specific math-modeling language
- APIs for C++, Python etc.
- New cool feature: Native functions !!!

Code Example

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Use of the C++ API:

```
// Exactly one machine is selected per station

for (int i = 0; i < nbStations; i++) {
LSExpression nbMachinesSelected = MyModel.sum();
for (int j = 0; j < nbCandidateMachines[i]; j++){
nbMachinesSelected += X[i][j];
}
MyModel.constraint(nbMachinesSelected == 1);
}
```

Flow line decomposition in native function

Flow line decomposition:

- Each station s characterized by $\mathrm{E}[T_s]$ and c_s^2

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- Called by LocalSolver via API during during each LocalSolver search move

Branch&Bound: Relaxation

Relaxation of integrality constraints

- Stations mixed by fractions $0 \leq \overline{v}_{s,j} \leq 1$ of machines

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$E[T_s]$ and $Var[T_s]$ of stochastic virtual mixed processing times T_s

$$T_s = \sum_{j=1}^{J_s} \overline{v}_{s,j} \cdot T_{s,j}$$

Important assumption: perfect correlation between $T_{s,i}$ and $T_{s,j}$!!!!!

Branch & Bound: Lower bounds on required budget

Basic idea

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4. Iterate
    1. Phase II: Re-distribute current budget while PR increases
    2. Phase III: Decrease budget until $PR \approx PR^{min}$
5. Terminate when budget stops to decrease for feasible solution or when PR^{min} is not reached in Phase I

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Observation: Relaxed selection variables $\overline{v}_{s,j}$ often binary, buffer variables $\overline{x}_s$ never

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Steepest ascent method

1. PR highly non-linear, frequent gradient updates
2. Termination, numerical issues

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- Methods seems to work (in principle)
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Thank you!!