Performance evaluation of a two-machine line with a finite buffer and condition-based maintenance

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  ⋆ The maintenance literature has not focused on the effects of maintenance on manufacturing systems composed of machines separated by finite buffers.

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  - ⋆ This has made it possible to model production lines with machines that have multiple down states, with machines that have non-exponential up- and down-times, and with multiple machines in parallel.
  - ⋆ Here, we apply this new technology to model machines that deteriorate with use, and whose condition is improved with maintenance.

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- The last two are examples of *condition-based maintenance.* Recent advances in sensor and computer technology have made this possible and have reduced costs.
- Here, we use recent advances in machine models to incorporate condition-based maintenance in production line analysis.

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  - ⋆ The operator may perform maintenance when it is in a specified degraded state to bring it to a specified better state.
    - The *maintenance policy* is the state at which maintenance is started and the state that it goes to.
  - ⋆ Failures such as a burned motor or an interruption of supply of consumable are repaired immediately. These are *minimal repairs* and they return the machine to the same deteriorated operational state as before the failure.

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- The steady-state performance (production rate, maintenance cost) is a function of the maintenance policy.
- We can form a simple optimization problem: choose the maintenance policy that minimizes maintenance cost subject to a production rate constraint.

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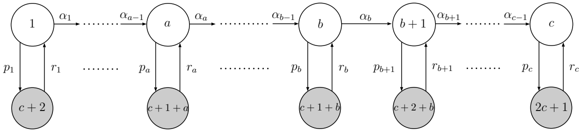
  - ⋆ The performance measures are the production rate, the maintenance cost, and the average inventory.
- We present numerical results.
- We discuss issues related to optimization, but we defer a full discussion of it.

Single Machine Model

Without Maintenance



Model of a deteriorating machine without maintenance

- Continuous time, discrete state, continuous material.
- States 1, ..., c are operational states.
- Processing rate in state i is $\mu_i \geq 0, i = 1, ..., 2c+1; \mu_i = 0, i > c$.
- Deterioration: transition rate from i to $i+1$ is $\alpha_i, i = 1, ..., c-1$.
- When the machine is in state i, it can go to state $c+1+i$ at rate p_i. This is a failure. The processing rate in $c+1+i$ is 0. It can go from $c+1+i$ to i at rate r_i (a minimal repair).

Without maintenance, the final class of states is c and $2c+1$.

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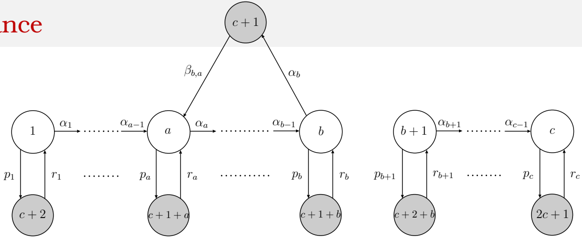
Future research will generalize the model to include more kinds of deterioration.

Single Machine Model

With Maintenance



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- The maintenance policy is (b, a).

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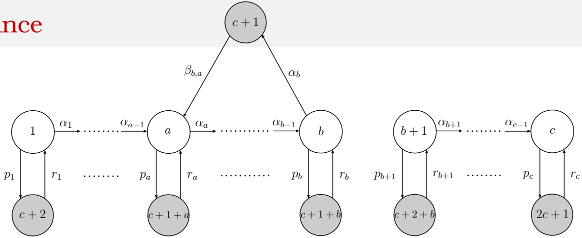

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- The maintenance policy is (b, a).
- When the machine is in state b, and a further deterioration occurs, the system is not allowed to go to state $b + 1$.

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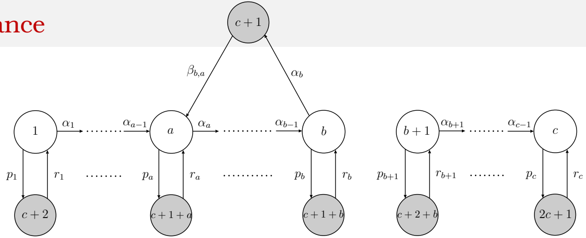

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- The maintenance policy is (b, a).
- When the machine is in state b, and a further deterioration occurs, the system is not allowed to go to state $b+1$.
- Instead, it goes to a new state, called $c+1$. This represents *a maintenance activity.* $\mu_{c+1} = 0$.

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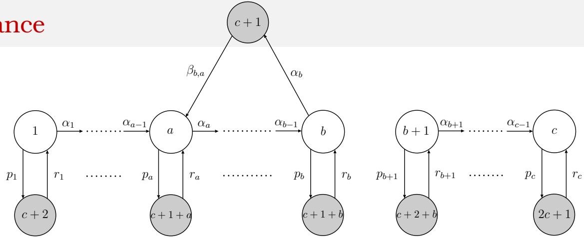

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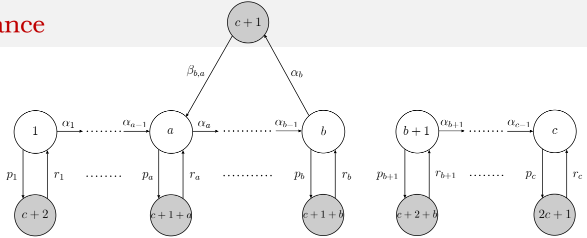

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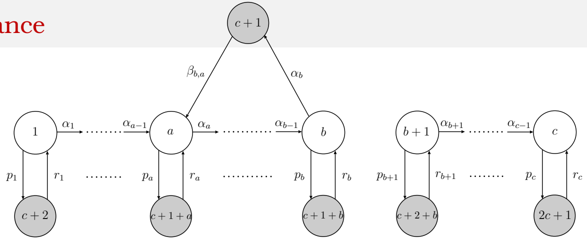

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The final class of states is $\{a, ..., b, c+1, c+1+a, ..., c+1+b\}$.

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Production rate

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- Production only occurs in operational states. Therefore the production rate is

$$\rho = \sum_{i=1}^{c} \mu_i P_i = \sum_{i=a}^{b} \mu_i P_i$$

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Production rate and the maintenance policy

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  - Production rate decreases with maintenance time and maintenance frequency.

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    - An (b, a) maintenance event that takes T time units costs $CPM_{ba}T$.
    - If the machine fails while it is in state i and the minimal repair takes T time units, the repair costs $CMR_i T$.
  - ⋆ Therefore, the total maintenance cost rate, the average steady-state rate of expenditure on minimal repairs and maintenance, is

$$TMC(b, a) = \sum_{i=1}^{b} P_{i+c+1} CMR_i + P_{c+1} CPM_{ba}$$

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subject to

$$\rho = \sum_{i=a}^{b} \mu_i P_i \geq K$$

Numerical Example

Problem: Find a maintenance strategy to minimize total cost such that the production rate ρ of a machine is greater than or equal to .8. The machine has four operational states[1].

[1] Details of all examples (including parameter values) are in Fitouhi, Nourelfath, and Gershwin, *Reliability Engineering and System Safety*, 2017, available on line.

Numerical Example

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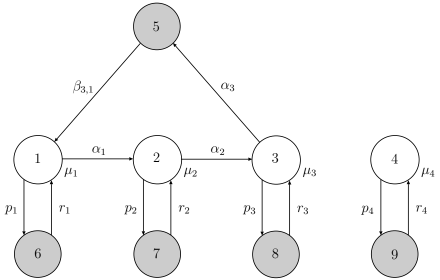
There are 10 possible policies: (1,1), (2,1), (2,2), ..., (4,1), (4,2), (4,3), (4,4).



Machine model with optimal solution

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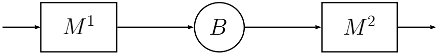
Numerical Example

b	a	Production rate	Minimal repair cost	Preventive maintenance cost	Total cost
1	1	0.9390	11.7371	56.3380	68.0751
2	1	0.8958	14.4635	50.3888	64.8523
2	2	0.7941	17.6471	70.5882	88.2353
3	1	0.8562	15.0024	46.9132	**61.9156**
3	2	0.7711	17.0790	57.5851	74.6641
3	3	0.7034	15.5172	68.9655	84.4828
4	1	0.7992	14.4684	68.2047	82.6730
4	2	0.7182	15.7562	66.8644	82.6206
4	3	0.6621	14.3956	61.5385	75.9341
4	4	0.5769	12.3077	61.5385	73.8462

Production rates and maintenance costs
for all possible maintenance strategies

Two-Machine, One-Buffer Model



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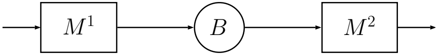

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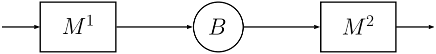

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- Notation: same as before, but each parameter and variable has a superscript indicating the machine.

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$$\longrightarrow \boxed{M^1} \longrightarrow \textcircled{B} \longrightarrow \boxed{M^2} \longrightarrow$$

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- State: (x, i_1, i_2) in which x is the amount of material in the buffer and i_j is the state of machine j, $j = 1, 2$.

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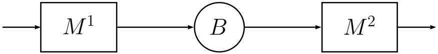

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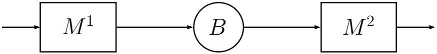

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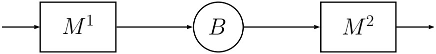
Two-Machine One-Buffer Model

x dynamics



- If the buffer is neither empty nor full ($0 < x < N$),

$$\frac{dx}{dt} = \mu^1_{i_1} - \mu^2_{i_2}$$

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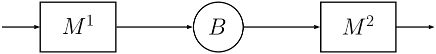

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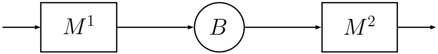
- If the buffer is empty($x = 0$), $\mu^2_{i_2} > 0$, and $\mu^1_{i_1} - \mu^2_{i_2} < 0$ then

$$\frac{dx}{dt} = 0 \text{ and } p^2_{i_2} \text{ and } \alpha^2_{i_2} \text{ are reduced.}$$

Two-Machine One-Buffer Model

x dynamics



- If the buffer is neither empty nor full $(0 < x < N)$,

$$\frac{dx}{dt} = \mu^1_{i_1} - \mu^2_{i_2}$$

- If the buffer is empty$(x = 0)$, $\mu^2_{i_2} > 0$, and $\mu^1_{i_1} - \mu^2_{i_2} < 0$ then

$$\frac{dx}{dt} = 0 \text{ and } p^2_{i_2} \text{ and } \alpha^2_{i_2} \text{ are reduced.}$$

- If the buffer is full $(x = N)$, $\mu^1_{i_1} > 0$, and $\mu^1_{i_1} - \mu^2_{i_2} > 0$ then

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- We use this methodology to evaluate the system we described in this presentation.

Performance Measures

Production rate

The rate of flow of material into the first machine is

$$\Pi = \sum_{(i_1,i_2)\in S_0} \mu^1_{i_1} \mathbf{P}(0, i_1, i_2) + \sum_{(i_1,i_2)\in S_M} \int_0^N \mu^1_{i_1} f(x, i_1, i_2) dx + \sum_{(i_1,i_2)\in S_N} \mu^2_{i_2} \mathbf{P}(N, i_1, i_2)$$

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Average buffer level:

$$\overline{x} = \sum_{i_1=1}^{(2c_1+1)} \sum_{i_2=1}^{(2c_2+1)} \left(\int_0^N x f(x, i_1, i_2) dx + N\mathbf{P}(N, i_1, i_2) \right)$$

Performance Measures

The steady-state probability of machine 1 being in state i_1 is

$$P^1_{i_1} = \sum_{i_2=1}^{2c_2+1} \left(\int_0^N f(x, i_1, i_2) dx + \mathbf{P}(0, i_1, i_2) + \mathbf{P}(N, i_1, i_2) \right),$$

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The steady-state probability of machine 2 being in state i_2 is

$$P^2_{i_2} = \sum_{i_1=1}^{2c_1+1} \left(\int_0^N f(x, i_1, i_2) dx + \mathbf{P}(0, i_1, i_2) + \mathbf{P}(N, i_1, i_2) \right),$$

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Performance Measures

Costs

The expected **minimal repair cost** for machine j is

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The expected **preventive maintenance cost** for machine j subject to maintenance policy (b_j, a_j) is:

$$CPM^j = CPM^j_{b_j,a_j} P^j_{c_j+1}$$

$j = 1, 2$

Performance Measures

Costs

The **total maintenance cost** is:

$$TMC = \sum_{i_1=1}^{c_1} \left(CMR^1_{i_1} P^1_{i_1+c_1+1} \right) + CPM^1_{b_1,a_1} P^1_{c_1+1} + \sum_{i_2=1}^{c_2} \left(CMR^2_{i_2} P^2_{i_2+c_2+1} \right) + CPM^2_{b_2,a_2} P^2_{c_2+1}$$

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The **total cost** is

$$TC = TMC + TC_{inv}$$

Example 1 — Simple Optimization

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- In Example 1, the two machines are the same as the one in the single-machine example.
- The buffer size is $N = 20$.
- There are 100 possible combinations of maintenance policies of both machines. We evaluate all of them.
- Consider the problem of choosing the policy with the least total cost and with a production rate Π greater than or equal to $\Pi_{min} = .75$.

Example 1 — Simple Optimization

- On the next slide, we list all policies, and their performance measures, that have a production rate greater than or equal to $\Pi_{min} = .75$. (There are 26.)

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- The optimal policy is (3,2,1,1).
- Note that using the single-machine optimal policy for both machines (3,1,3,1) is 19% more expensive.
- Note also the difference in performance between (3,2,1,1) and (1,1,3,2). *It is better if the first machine is the bottleneck, if you have a choice.*

Example 1 — Simple Optimization

Machine M^1			Machine M^2			Two-machine line		
b_1	a_1	ρ^1	b_2	a_2	ρ^2	$\bar{x}$	Π	TC
1	1	0.9390	1	1	0.9390	10.0000	0.9095	181.872
1	1	0.9390	2	1	0.8958	13.0958	0.8744	192.177
2	1	0.8958	1	1	0.9390	6.9042	0.8744	161.220
2	1	0.8958	2	1	0.8958	10.0000	0.8507	173.173
3	1	0.8562	1	1	0.9390	5.4381	0.8390	150.125
1	1	0.9390	3	1	0.8562	14.5619	0.8390	195.744
3	1	0.8562	2	1	0.8958	8.0210	0.8217	160.421
2	1	0.8958	3	1	0.8562	11.9790	0.8217	180.211
3	1	0.8562	3	1	0.8562	10.0000	0.7986	168.246
4	1	0.7992	1	1	0.9390	4.4390	0.7856	162.191
1	1	0.9390	4	1	0.7992	15.5610	0.7856	217.801
2	2	0.7941	1	1	0.9390	2.1521	0.7846	154.818
1	1	0.9390	2	2	0.7941	17.8479	0.7846	233.297
2	2	0.7941	2	1	0.8958	3.7451	0.7801	161.880
2	1	0.8958	2	2	0.7941	16.2549	0.7801	224.430
4	1	0.7992	2	1	0.8958	6.5522	0.7727	170.379
2	1	0.8958	4	1	0.7992	13.4478	0.7727	204.857
2	2	0.7941	3	1	0.8562	6.0344	0.7658	171.945
3	1	0.8562	2	2	0.7941	13.9656	0.7658	211.601
3	**2**	**0.7711**	**1**	**1**	**0.9390**	**1.8563**	**0.7626**	**140.924**
1	1	0.9390	3	2	0.7711	18.1437	0.7626	222.361
3	2	0.7711	2	1	0.8958	3.0820	0.7594	146.417
2	1	0.8958	3	2	0.7711	16.9180	0.7594	215.597
3	1	0.8562	4	1	0.7992	11.7171	0.7551	194.299
4	1	0.7992	3	1	0.8562	8.2829	0.7551	177.128
2	2	0.7941	2	2	0.7941	10.0000	0.7502	216.705

$N = 20$. Possible maintenance strategies with $\Pi \geq \Pi_{min} = 0.75$.

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- Observations:
  - ⋆ The first machine is the bottleneck for all Π_{min} except for the highest value (in which the line is balanced).
  - ⋆ The cost increases with Π_{min}.

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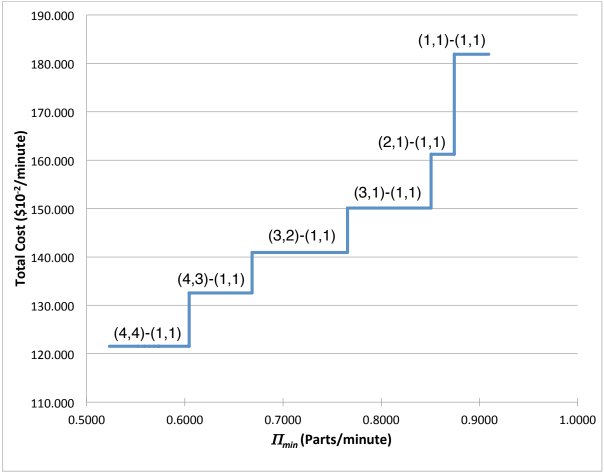

Variation of total cost associated with the best maintenance strategy

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- The policy changes at $N = 10$ to reduce the growth of inventory as the buffer size grows. This reduces the production rate of the line.
- *The production rate at* $N = 20$ *is less than that at* $N = 6$*!!*

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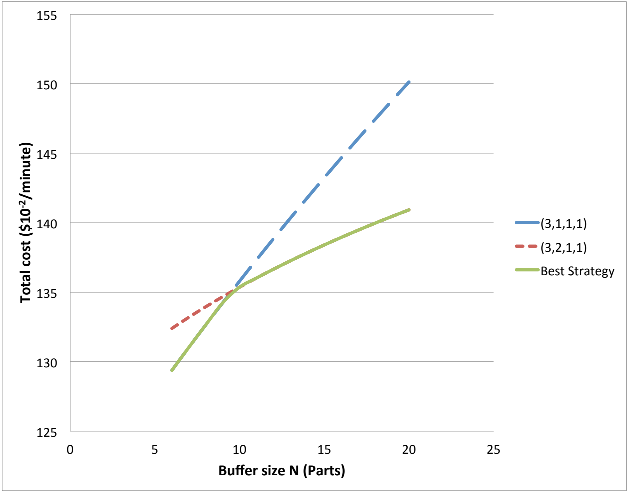

Total cost vs. buffer size

Example 4 — Varying Inventory Holding Cost

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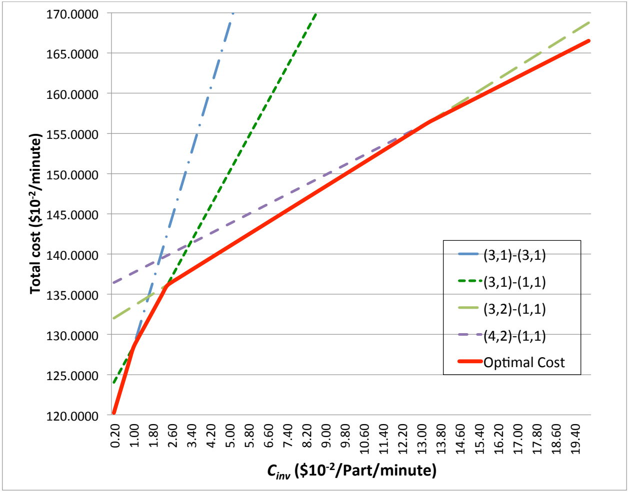

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- Inventory can be reduced by speeding up the second machine (increasing ρ_2) and slowing down the first machine (decreasing ρ_1).

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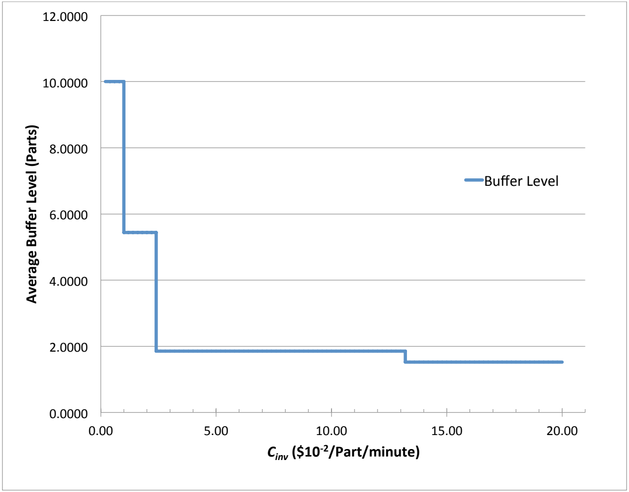

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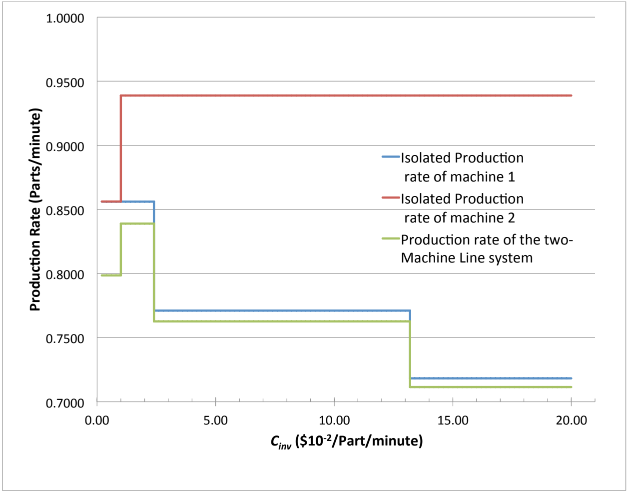

Impact of inventory cost on machine and system production rates

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- This means that we must modify the optimization problem formulation for it to be meaningful. The modified problem will have more decision variables.

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- Instead, we use the speed at which the machine is operating as a decision variable.
- Define $u^j_{i_j}$ to be the processing rate of machine j while it is in degradation state i_j.
- Then $0 \leq u^j_{i_j} \leq \mu^j_{i_j}$.

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All variables, parameters, and constraints defined previously are also part of the optimization problem.

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- We have performed numerical experiments and described observations of its qualitative behavior.
- We have proposed an initial version of an optimization problem to determine simultaneously the maintenance policy, the buffer size, and the speed of operating the machines.

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Thank you.

