Stability Conditions for Multiclass Queueing Networks

Kan Wu, Yichi Shen

Presented by Yichi Shen

yshen006@e.ntu.edu.sg

NANYANG TECHNOLOGICAL UNIVERSITY

Stability of Queueing Networks

- Jackson (1957)
  - Product-form solutions of Jackson networks.
- Lu and Kumar (1991)
  - A general network can be unstable even if the usual traffic condition is satisfied.
- Dai (1995)
  - A queueing network is stable if its corresponding fluid model is stable.

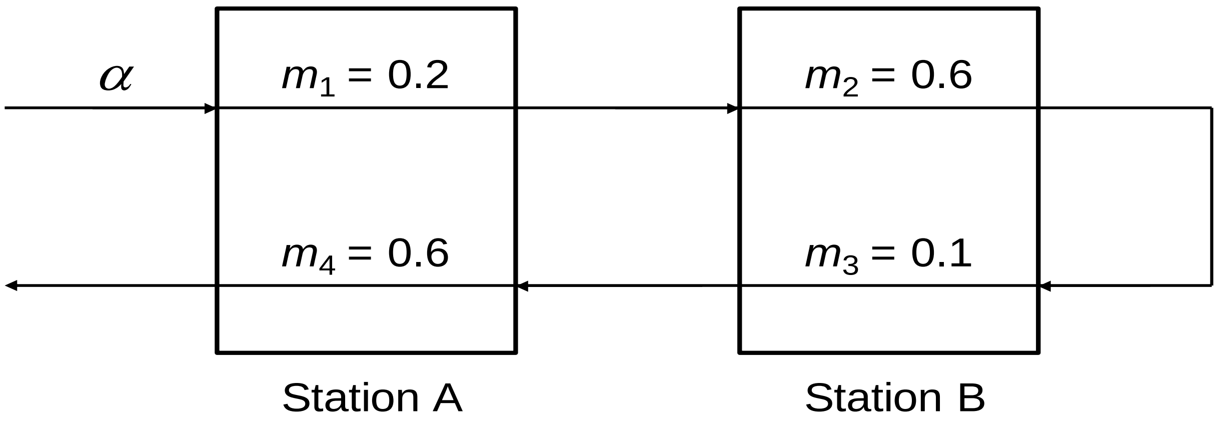
NANYANG TECHNOLOGICAL UNIVERSITY

Outline

- An Unstable Lu-Kumar Network
- Mutual Blocking
- General Servers in Queueing Networks
- Stability Conditions for Multiclass Queueing Networks

NANYANG TECHNOLOGICAL UNIVERSITY

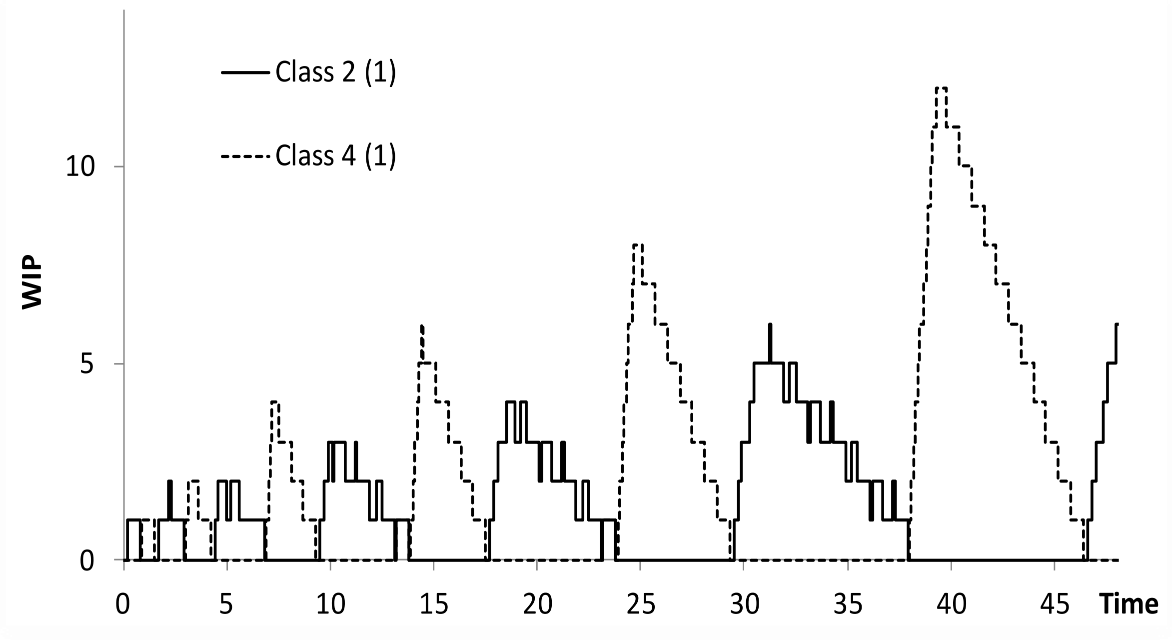
A Lu-Kumar Network



- $\alpha = 1, \rho_A = 0.8, \rho_B = 0.7.$
- Priority: 1<4, 2>3. Non-preemptive.

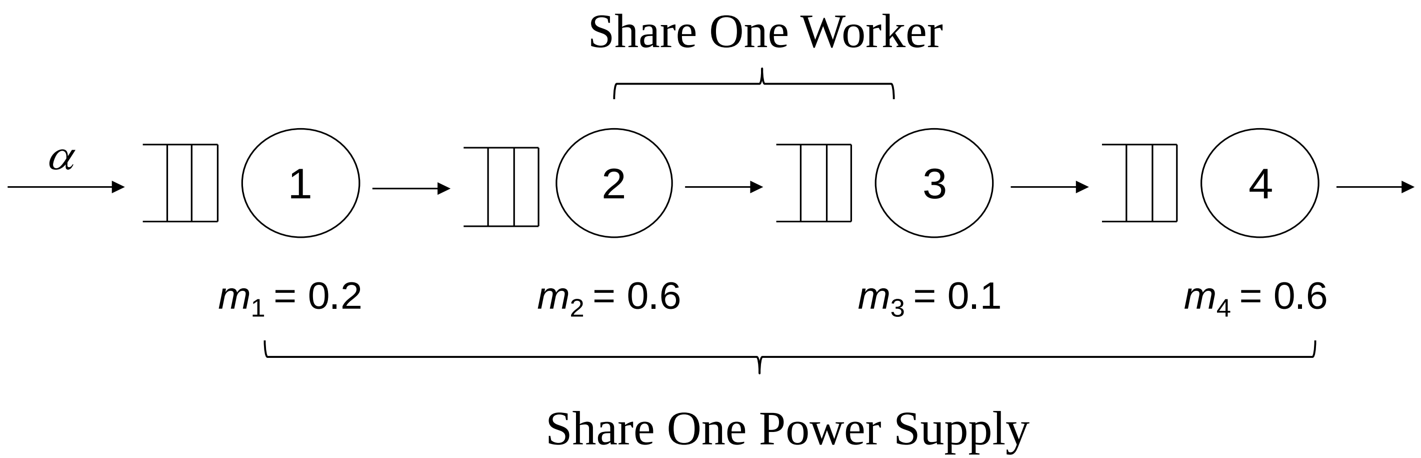
NANYANG TECHNOLOGICAL UNIVERSITY

WIP at Classes 2 and 4



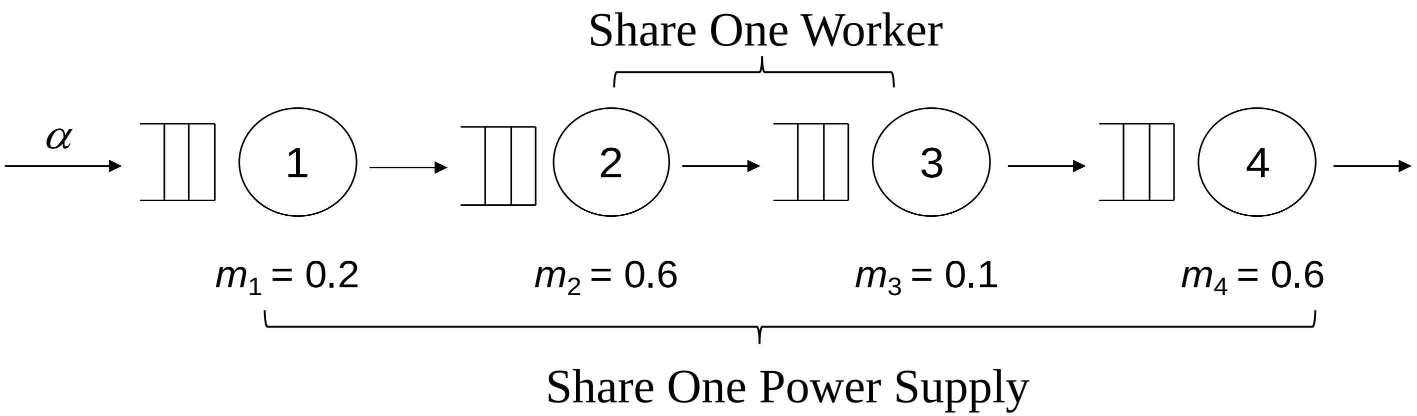
- The system is unstable although $\rho_A < 1$, ρ_B<1!
  - ***The usual traffic condition is not sufficient for stability!***

A Tandem Queue

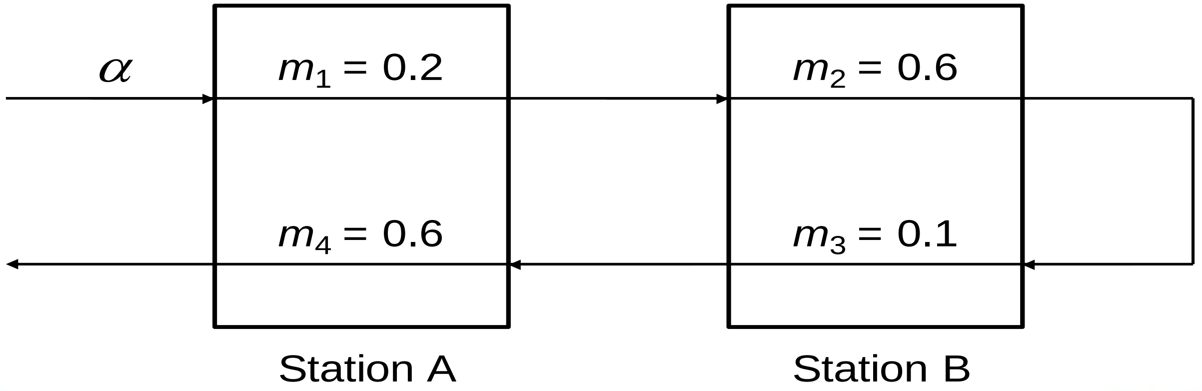


- Priority: 1<4, 2>3. Non-preemptive.

Equivalence of the Two Networks



VS



NANYANG TECHNOLOGICAL UNIVERSITY

Rethinking the Meaning of Servers…

- Should servers be defined based on the physical configurations in queueing networks?
  - The essence of physical stations is that at most one class at the station can receive service at any time…
  - The usual traffic condition is sufficient for single server queues. Why it fails when considering networks?
  - The meaning of servers may be different in the context of networks…

NANYANG TECHNOLOGICAL UNIVERSITY

Mutual Blocking

- Mutual Blocking
  - Under a given dispatching policy, the classes suffer mutual blocking if they cannot receive service simultaneously.

A class set $\mathcal{C}$ suffers mutual blocking if the following two conditions are satisfied almost surely:

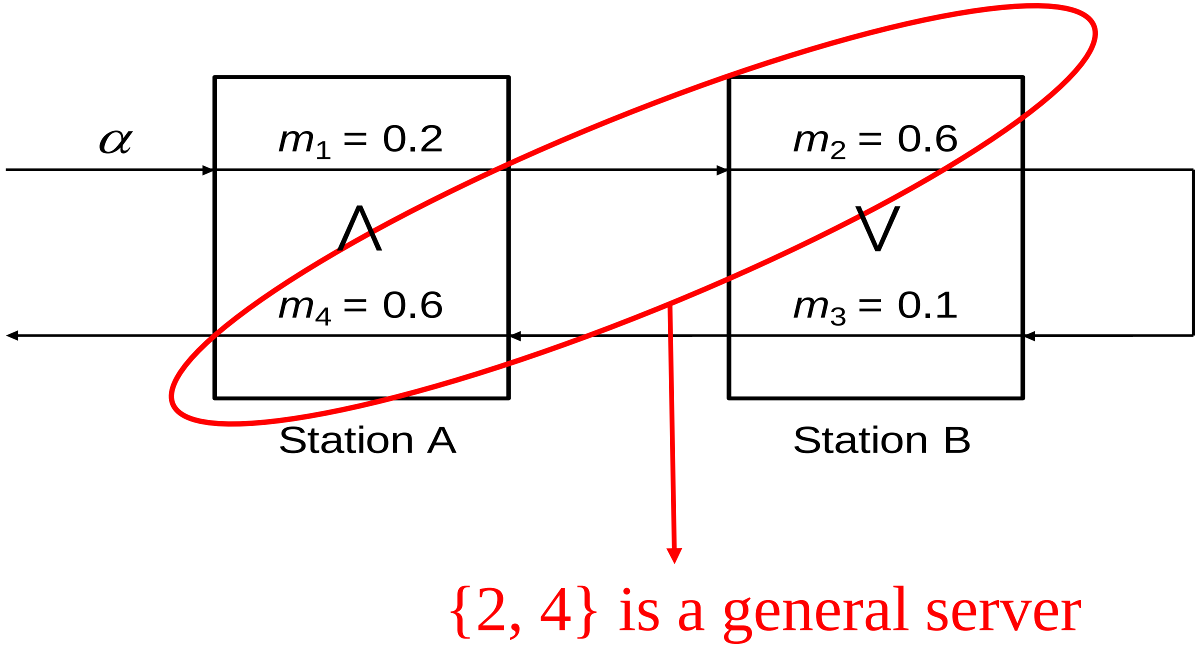
(a) $\lim_{t\to\infty} m\left(\{t \,|\, \prod_{i\in\mathcal{C}} \dot{R}_i(t) \neq 0\}\right) / m\left(\{t \,|\, \sum_{i\in\mathcal{C}} \dot{R}_i(t) \neq 0\}\right) = 0$, and

(b) If $\sum_{i\in\mathcal{C}} m_i/\alpha_i < 1$, then $\lim_{t\to\infty} m(\{t \,|\, \sum_{i\in\mathcal{C}} R_i(t) = 0\})/t < 1 - (\sum_{i\in\mathcal{C}} m_i/\alpha_i)$.

NANYANG TECHNOLOGICAL UNIVERSITY

General Servers

- Any class set suffering mutual blocking is a *general server*.



Effective Classes of a General Server

- Effective Number of Classes
  - While at most one class can receive service at any time in a physical station, a general server can have *M effective* classes.

For a general server S, its effective number of classes is M (denoted as $EF(S) = M$) if the following two conditions are satisfied almost surely:

(a) If $I \subseteq S$ and $|I| \geq M + 1$, then

$\lim_{t\to\infty} m\left(\left\{t \middle| \prod_{i\in I} \dot{R}_i(t) \neq 0\right\}\right) / m\left(\left\{t \middle| \sum_{i\in S} \dot{R}_i(t) \neq 0\right\}\right) = 0$, and

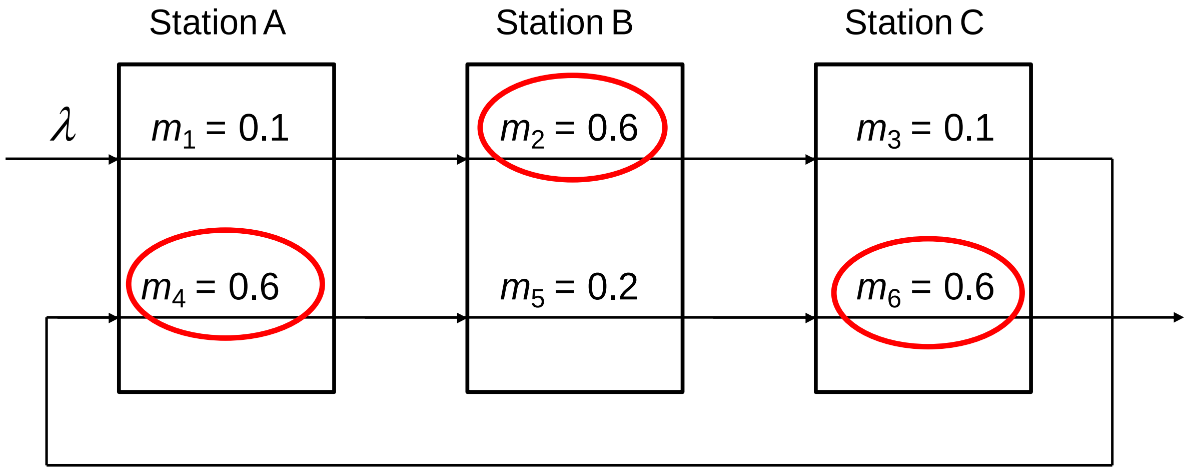
(b) There exists $I \subseteq S$ and $|I| = M$, s.t.

$\lim_{t\to\infty} m\left(\left\{t \middle| \prod_{i\in I} \dot{R}_i(t) \neq 0\right\}\right) / m\left(\left\{t \middle| \sum_{i\in S} \dot{R}_i(t) \neq 0\right\}\right) > 0$.

NANYANG TECHNOLOGICAL UNIVERSITY

General Servers

- Hasenbein (1997)
  - A six-class network. Priority: 1<4; 2>5; 3<6. Preemptive.



General Server {2, 4, 6}, E({2, 4, 6})= 2

NANYANG TECHNOLOGICAL UNIVERSITY

Traffic Intensity of a General Server

	Physical Stations	General Server
Effective Number	1	M
Load	$L = \sum_{k:\, \sigma(k)=j} \lambda_{\tau(k)} m_k$	$L_S = \sum_{k \in S} \lambda_{\tau(k)} m_k$
Traffic Intensity	$\rho = \sum_{k:\, \sigma(k)=j} \lambda_{\tau(k)} m_k \,/1$	$P_S = \sum_{k \in S} \lambda_{\tau(k)} m_k \,/M$

NANYANG TECHNOLOGICAL UNIVERSITY

Stability Conditions for Queueing Networks

- Stability Conditions
  - Under a given dispatching policy, a queueing network is pathwise stable if and only if the effective traffic intensity of every general server does not exceed one, *i.e.*,$\forall S \in \mathcal{S}$, $P_S \leq 1$.
  - The usual traffic condition becomes sufficient if all general servers are considered.

Instability of the Lu-Kumar Network

- The traffic intensity of general server{2, 4}
  - $\rho_{\{2,4\}} = 1.2 > 1$.
  - Server {2, 4} does not have sufficient capacity.
  - System is unstable.

NANYANG TECHNOLOGICAL UNIVERSITY

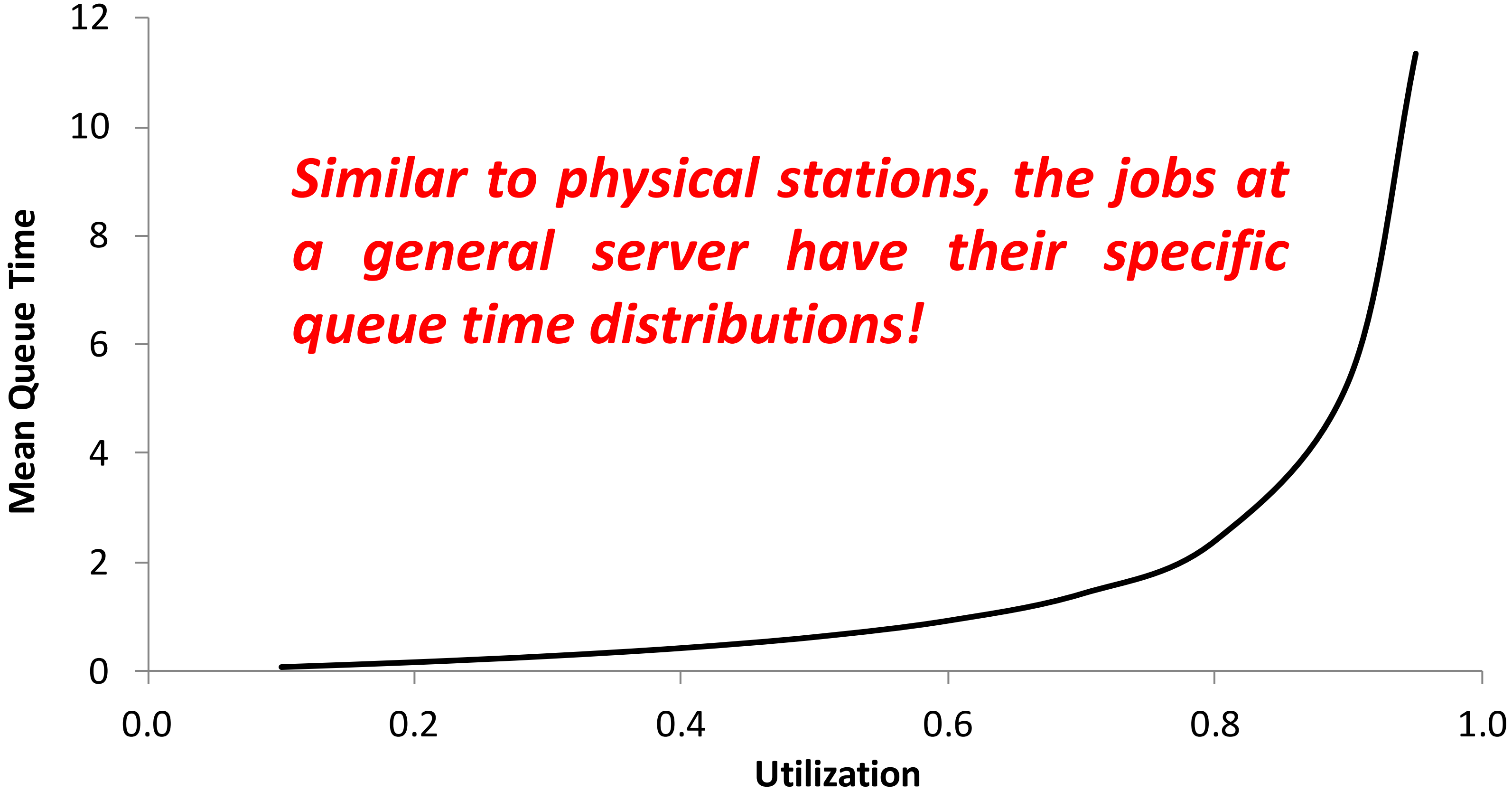
Sensitivity of General Servers

- Is {2, 4} a general server in the previous Lu-Kumar network?

	Non-Preemptive	Preemptive
$\alpha = 0.85$	×	√
$\alpha = 1$	√	√
$\alpha \sim U(0.8, 0.9)$	×	√
$\alpha \sim \exp(0.85)$	√	√

NANYANG TECHNOLOGICAL UNIVERSITY

Queue Times at a General Server



Job mean queue time at server $\{2, 4\}$ in the Lu-Kumar network

NANYANG TECHNOLOGICAL UNIVERSITY

Conclusion

- Focusing on physical stations is not enough when analyzing queueing networks.
- General servers dominate the stability of queueing networks.
- The structure of general servers is sensitive to the system configuration.
- Jobs at general servers have their specific queue time distributions.

Future Research

- General Servers
  - Develop general algorithms to find general servers.
  - Determine the effective number of classes.

- Queue times
  - Study the queue time distributions for general servers.

- Stability Type
  - Positive Harris Recurrence.

NANYANG TECHNOLOGICAL UNIVERSITY

Thank You!

Any Questions?

Dai, J. (1995). "On positive Harris recurrence of multiclass queueing networks: a unified approach via fluid limit models". *The Annals of Applied Probability*, 49-77.

Hasenbein, J.J. (1997). "Necessary conditions for global stability of multiclass queueing networks. *Operations Research Letters*, **21** (2): 87-94.

Jackson, J.R. (1957). "Networks of Waiting Lines". *Operations Research*, **5** (4): 518–521.

Lu, S.H. and P. Kumar (1991). "Distributed scheduling based on due dates and buffer priorities". *IEEE Transactions on Automatic Control*, **36** (12): 1406-1416.