Analysis of Performance Approximations for Queueing Networks with Non-Homogeneous Arrival Processes

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Dynamic Queueing Network Analysis
Outline

- Introduction
- Problem Definition
- Impact of Non-stationarity
- Literature Review
- Research Methodology
- Results
- Conclusions and Future Research
Introduction

- The manufacturing facility as a dynamic queueing network
Randomness, Certainty, Variability

Random: $\mu$

Uncertain (Bayesian): $f(\mu)$

Variable (Dynamic):
- $\mu(t_0)$
- $\mu(t_1)$
- $\mu(t_2)$
- $\mu(t_3)$
Introduction

- Often systems may not adhere to product form model assumptions
Introduction

- Rough cut planning and scenario analysis under dynamic conditions call for efficient algorithms.

- Research question: How do we leverage available efficient algorithms for steady-state analysis to develop reliable approximations for first (and maybe higher) order estimates of performance measures under dynamic conditions?
Problem Definition

- Consider an interconnected system of servers
- Non-stationary arrivals, Markovian service processes (possibly intermittent or non-homogenous) and FCFS queueing discipline.
- \( L \) workstations and \( R \) routing chains (customer classes/part types).
- Infinite queue capacity and no migration.
- External and internal arrivals governed by an irreducible stochastic routing matrix, \( P_r \) (i.e. process plans by part type)
- State of the system can be described by state vector
  \[
  \mathbf{K} = (\mathbf{k}_1, \ldots, \mathbf{k}_R) \quad \text{where} \quad \mathbf{k}_r = (k_{r,1}, \ldots, k_{r,L})
  \]
  where \( k_{r,l} \): number of class \( r \) jobs at workstation \( l \)
Impact of Non-stationarity

- Example 1: Three stage serial production line.
Impact of Non-stationarity

- Example 2: Three stage flow shop.

Arrival Process: Complementary Linear Non-Homogenous Poisson Processes
Literature Review : Nonstationary Queues

Some general results e. g. Heyman and Whitt (1984): dynamic steady state for $M_t/G/c$

Three broad categories of approximations

• Systems Approximations (period by period stationary approximation)
  • Green and Kolesar (1991): Pointwise stationary approximation (PSA)
  • Green et al. (2001): Stationary independent period-by-period (SIPP) approximation
  • Stolletz (2008): Stationary backlog carryover (SBC) approximation

• Numerical Approximations (Simplification assumptions and numerically compute)
  • Rothkopf and Oren (1979): Closure approximation
  • Grassmann (1977): Randomization method

• Process Approximations (Limiting heavy traffic fluid/diffusion methods)
  • Mandelbaum and Massey (1995)
  • Wang et al. (1996): Pointwise stationary fluid flow approximation (PSFFA)

Ingolfsson et al. (2007): Experimental comparison of seven service-level approximations for nonstationary queues.
Literature Review: Nonstationary Queueing Networks

Duda (1986): Parametric decomposition based on transient analysis of $GI/GI/1$ queue


Mandelbaum and Massey (1995): Fluid and diffusion limits for large scale Markovian service networks

Whitt (1999): Generalized Jackson network based approximation framework for time-dependent Markovian networks-simplifying system of ODEs

Motivation

• Research focus so far has been on
  – Nonstationary queues
  – Queuing networks under stationary arrival assumptions or special conditions

• Need efficient algorithms for QN’s under dynamic conditions.

• Incorporating network structure enables
  – Understanding evolution of congestion at different points
  – Modelling class priorities

• Focus is on first-order estimates of system performance.
System Approximation Research Methodology

• Total observation window $T$ is broken down into a finite number of time epochs of equal length $t_s$, $s=1,..,\lfloor T/t_s \rfloor$.

• System dynamics are studied through snapshots of system performance tracked for each time epoch.

• Snapshots are a weighted combination of steady state performance metrics for two closed queueing networks, with the floor and ceiling levels of WIP (Basic Closed Model).
Research Methodology

• Steady state estimates are precomputed using an exact Mean Value Analysis (MVA) algorithm (see Reiser and Lavenberg (1980)).

• Characteristics of MVA
  – Provides steady state estimates for all intermediate levels of WIP (provides performance for all values of \( k = 1,..,K_r \))
  – Complexity: \( O(\prod_{r=1}^{K_r}) \), where \( K_r \) is number of jobs of class ‘r’.
Research Methodology

- **Step 1.** Solve closed network, $C(N^*)$ using the MVA algorithm. The state vector $N^*$ is set to a sufficiently large value.

- **Step 2.** Initialize $i=0$, $t=0$ and the vector of total jobs in each routing chain for initial conditions, $N_0 = (n_1(0), n_2(0), ..., n_R(0))$
Set $X_r(0) = X^c_r(N_0)$
Step 3. Set $i = i + 1$, $t = t + t_s$.

Update mean number in routing chain $r = 1, ..., R$ as

$$n_r(t) = \max(n_r(t - t_s) + t_s \sum_{j=1}^{L} \lambda_{rj}(t - t_s, t) - t_s \cdot X_r(t - t_s), 0)$$

(Starting + Arrivals – Completions; or 0)

If $t < T$ go to step 4 else STOP.
Research Methodology

- Step 4. For each $r = 1, \ldots, R$
  a) Update throughput rate as
  \[ X_r(t) = (1 - \alpha_r) X_r^c(N_l) + \alpha_r X_r^c(N_u) , \text{ where} \]
  \[ N_l = (\lfloor n_1(t) \rfloor, \lfloor n_2(t) \rfloor, \ldots, \lfloor n_R(t) \rfloor) , \quad \text{and} \quad \alpha_r = n_r(t) - \lfloor n_r(t) \rfloor \]
  b) Update mean total time in system as
  \[ W_r(t) = \frac{n_r(t)}{X_r(t)} \quad \text{(Little’s law for each routing chain)} \]
  c) Update Cumulative production as
  \[ \chi_r(t) = \chi_r(t - t_s) + \min(X_r(t) \cdot t_s, (n_r(t - t_s) + t_s \sum_{j=1}^{L} \lambda_{rj}(t - t_s, t))) \]
- Step 5. Go to step 3.
Computational Analysis

• Method pieces together snapshots of how a stationary system would perform at each time step with the given WIP level.

• Approximation MVA assumes distribution of jobs across workstations not representative of actual distribution for non-homogenous process.

• Approximation was applied to a simple serial system and a jobshop with four classes and sixteen workstations.

• First order moments of interest were compared against simulation results.
Experiments

- Models
  - 1a.
    - Type: Three workstation serial line
    - Arrival Process: Non – homogenous Sinusoidal Poisson Process with $2\pi = 1000 \text{ min}$.
  - 1b.
    - Type: Three workstation serial line
    - Arrival Process: Non – homogenous Sinusoidal Poisson Process with $2\pi = 100 \text{ min}$.
  - 2.
    - Type: Two class three workstation flow shop
    - Arrival Processes: Complementary non-homogenous linear Poisson process
  - 3.
    - Type: Two class three workstation flow shop
    - Arrival Processes: Complementary non-Homogenous sinusoidal Poisson process.
Model 1a: Time in system

\[ 2\pi = 1000 \text{ min}. \]
Cumulative production results

$2\pi = 1000 \text{ min}.$

Model 1a

Model 2
Model 1b: Time in system

$2\pi = 100\text{ min.}$

Note: Static model gives a constant time in system of 15.56 mins
Model 1b: Cumulative production

\[ 2\pi = 100 \text{ min}. \]
Model 1b: Absolute Deviation

$2\pi = 100 \text{ min}$.

Mean % Absolute Deviation Approx. vs Simulation

Absolute Deviation

Cumulative Production vs Time in System

Mean % Absolute Deviation

Cumulative Production for Serial Line
Model 2: Absolute Deviation

**Mean % Absolute Deviation Approx. vs Simulation for Product 1**

- Number in System:
  - 5 min: 45.84
  - 20 min: 43.09
  - Static Model: 266.56

- Cumulative Production:
  - 5 min: 17.04
  - 20 min: 18.16
  - Static Model: 694.19

**Mean % Absolute Deviation Approx. vs Simulation for Product 2**

- Number in System:
  - 5 min: 33.96
  - 20 min: 35.14
  - Static Model: 50.24

- Cumulative Production:
  - 5 min: 2.07
  - 20 min: 2.05
  - Static Model: 27.31
Model 3: Absolute Deviation

Mean % Absolute Deviation
Approx. vs Simulation for Product 1

Mean % Absolute Deviation
Approx. vs Simulation for Product 2

Number in System  | Cumulative Production
--- | ---
35.98  | 80.4
1.96  | 1.2
12.02

Number in System  | Cumulative Production
--- | ---
18.81  | 60.59
1.21  | 1.53
9.56
Enhancements to Base model: Motivation

• Implicit Dynamic distribution of jobs ignored in previous model.

• Steady state assumptions don’t hold under transient conditions.

• Instantaneous throughput rates of individual workstations need not be identical.
Route Workstation Based Throughput Model (RWBTM)

- **Step 1**
  Use MVA to solve the closed queueing network $C(N^*)$ ($N^*$ sufficiently large).

- **Step 2**
  Initialize $i = 0$, $t = 0$.
  Set $N_0 = (n_1(0), \ldots, n_R(0))$ to match an equivalent open queueing network.
  Set $X_r(0) = X_r^c(N_0)$ for each $r = 1, \ldots, R$.
  Set $n_{rl}(0) = n_{rl}^c(N_0)$, $\gamma_r l(0) = 0$, $\psi_{rl}(0) = 0$, $X_{rl}(0) = \nu_{rl} X_r^c(N_0)$ for each $r = 1, \ldots, R$ and each $l \in S(r)$.

- **Step 3**
  Set $t = t + t_s$
Route Workstation Based Throughput Model (RWBTM)

• Step 4

For each $r = 1, \ldots, R$ and each $l \in S(r)$

1. Compute mean number of jobs of routing chain $r$ leaving workstation $l$ at time $t$

$$\psi_{rl}(t) = \min(n_{rl}(t - t_s) + t_s \lambda_{rl}(t - t_s, t), t_s X_{rl}(t - t_s))$$

2. Compute total internal arrivals of routing chain $r$ at workstation $l$ at time $t$

$$\gamma_{rl}(t) = \sum_{j=1}^{L} \psi_{rj}(t)p_{jl}^{r}$$
Route Workstation Based Throughput Model (RWBTM)

- Step 5

Update cumulative production at time $t$ for each $r = 1, \ldots, R$

$$\chi_r(t) = \chi_r(t - t_s) + \sum_{l=1}^{L} \left[ \left( 1 - \sum_{j=1}^{L} p_{lj}^{r} \right) \psi_{rl}(t) \right]$$
Route Workstation Based Throughput Model (RWBTM)

• Step 6

For each $r = 1, ..., R$ and $l \in S(r)$

1. Update mean number in routing chain $r$ at workstation $l$ as

$$n_{rl}(t) = n_{rl}(t - t_s) + t_s \lambda_{rl}(t - t_s, t) + \gamma_{rl}(t) - \psi_{rl}(t)$$  \hspace{1cm} (1)

2. Obtain closest CQN, $N_l^*(t)$ for each workstation $l$ as

$$N_l^*(t) = \{ N : n_l^*(t) = \min_{N \in N^*} ||n_l(t) - n_l^c(N)||_2 \}$$ \hspace{1cm} (2)

3. Update throughput rate for routing chain $r$ at workstation $l$ as

$$X_{rl}(t) = \nu_{rl}X_r^c(N_l^*(t))$$ \hspace{1cm} (3)

• Step 7

If $t < T$ GO TO 3. Else STOP.
Experiments

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  - 3.
    - Type: Two class three workstation flow shop
    - Arrival Processes: Complementary non-Homogenous sinusoidal Poisson process.
RWBTM: Model 1b WIP

- WIP at Workstation 1
- WIP at Workstation 2
- WIP at Workstation 3
Model 1b: Cumulative production
Model 2: Work - in - Process
Model 2: Cumulative production
Model 3: Work - in - Process
Model 3: Cumulative production
Model 1a: Absolute Deviation

Mean % Absolute Deviation Approx. vs Simulation - WIP

Mean % Absolute Deviation Approx. vs Simulation

Cumulative Production

<table>
<thead>
<tr>
<th>Time</th>
<th>WIP at W/S 1</th>
<th>WIP at W/S 2</th>
<th>WIP at W/S 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5 min</td>
<td>Blue</td>
<td>Blue</td>
<td>Blue</td>
</tr>
<tr>
<td>5 min</td>
<td>Red</td>
<td>Red</td>
<td>Red</td>
</tr>
<tr>
<td>20 min</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time</th>
<th>Cumulative Production</th>
</tr>
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<tbody>
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<td>0.5 min</td>
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<tr>
<td>5 min</td>
<td>Red</td>
</tr>
<tr>
<td>20 min</td>
<td>Green</td>
</tr>
</tbody>
</table>
Model 2: Absolute Deviation

Mean % Absolute Deviation Approx. vs Simulation - WIP

Titolo del grafico
Model 3: Absolute Deviation

Mean % Absolute Deviation Approx. vs Simulation - WIP

Mean % Absolute Deviation Approx. vs Simulation

Route 1 Cumulative Production

Route 2 Cumulative Production
Open Network Based Throughout Model (ONBTM)

\( \lambda_{ij}^r(t), \ r \in \{0, \ldots, R\}, \ i \in \{0, \ldots, L\}, \ j \in \{1, \ldots, L\} \): rate of class \( r \) arrivals at station \( j \) from station \( i \) (station 0 for external arrivals).

- **Step 1**
  Initialize \( n_{rl} \).

- **Step 2**
  Update throughput for class \( r \) at workstation \( l \) for each \( r = 1, \ldots, R \) and \( l \in S(r) \setminus \{0\} \)

\[
X_{rl}(t) = \min \left[ \left( \frac{n_{rl}(t)}{\sum_{p=1}^{R} n_{pl}(t) + 1} \right) \mu_{rl}, \left( \frac{n_{rl}(t) + \frac{t_s \lambda_{0l}^r(t)}{2}}{t_s} \right) \right]
\]

Min of (Effective allocated production resource; Available WIP)
Open Network Based Throughout Model (ONBTM)

• Step 3
For each $r = 1, \ldots, R$ and $l \in S(r) \setminus \{0\}$ (0 is external)

Update arrival rates of class $r$ arriving at workstation $l$ from workstation $k$

$$\lambda_{kl}^r(t) = p_{kl}^r X_{rk}(t), \; k \in S(r) \setminus \{0\}$$

• Step 4
Update WIP for each $r = 1, \ldots, R$ and $l \in S(r) \setminus \{0\}$

$$n_{rl}(t + t_s) = \max \left[ n_{rl}(t) + t_s \left( \sum_{j=0}^{L} \lambda_{jl}^r(t) - \sum_{j=0}^{L} \lambda_{lj}^r(t) \right), 0 \right]$$

• Step 5
Set $t = t + t_s$. If $t < T$ GO TO 2. Else STOP.
Experiments

- Models
  - 1a.
    - Type: Three workstation serial line
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  - 1b.
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ONBTKM Results: Model 1a Work - in - Process
Model 1a: Cumulative production
Model 1b : Work-in-Process

- WIP at Workstation 1
- WIP at Workstation 2
- WIP at Workstation 3
Model 1b: Cumulative production
Model 2: Work - in - Process

Class 1 WIP at Workstation 1

Class 1 WIP at Workstation 2

Class 1 WIP at Workstation 3

Class 2 WIP at Workstation 1

Class 2 WIP at Workstation 2

Class 2 WIP at Workstation 3
Model 2: Cumulative production
Model 3: Work-in-Process

Class 1 WIP at Workstation 1

Class 1 WIP at Workstation 2

Class 1 WIP at Workstation 3

Class 2 WIP at Workstation 1

Class 2 WIP at Workstation 2

Class 2 WIP at Workstation 3
Model 3: Cumulative production
Experiment: “Large” Jobshop Instance

- Jobshop instance with four classes (1,...,4) and sixteen workstations (1,...,16) (bottleneck: workstation 4)

<table>
<thead>
<tr>
<th>Product</th>
<th>Routing</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4 → 12 → 2 → 5 → 13 → 8</td>
</tr>
<tr>
<td>2</td>
<td>2 → 3 → 1 → 4 → 6 → 7 → 5 → 9 → 10 → 11 → 12 → 13 → 14 → 15 → 16</td>
</tr>
<tr>
<td>3</td>
<td>1 → 8 → 10 → 9 → 11 → 13 → 14 → 4 → 16</td>
</tr>
<tr>
<td>4</td>
<td>4 → 5 → 3 → 8 → 10 → 6 → 9 → 12 → 1 → 15 → 16</td>
</tr>
</tbody>
</table>

Table 1: Jobshop Routing

- Basic Model and ONBTM estimates compared against thousand simulation replications
- Two non-homogenous arrival patterns were investigated
  - Peak shifted triangular pattern with peak offset = 200 mins. (Pattern 2)
  - Phase shifted sinusoidal with $2\pi = 100 \text{ mins}$. (Pattern 1)
Basic Model: Cumulative Production

Pattern 1

Pattern 2
Basic Model: Class 1 WIP

Pattern 1

Pattern 2
Basic Model: Class 2 WIP

Pattern 1

Pattern 2
Basic Model: Class 3 WIP

Pattern 1

Pattern 2
Basic Model: Class 4 WIP

Pattern 1

Pattern 2
ONBTM: Cumulative Production

Pattern 1

Pattern 2
ONBTM: Class 1 WIP at Workstation 4

Pattern 1

Pattern 2
ONBTM: Class 2 WIP at Workstation 4

Pattern 1

Pattern 2
ONBTM: Class 3 WIP at Workstation 4

Pattern 1

Pattern 2
ONBTM: Class 4 WIP at Workstation 4

Pattern 1

Pattern 2
Large JobShop: Absolute Deviation

Basic Model: Mean % Absolute Deviation

ONBDM: Mean % Absolute Deviation

Total WIP in System

WIP at bottleneck station

Class 1  Class 2  Class 3  Class 4

Pattern 1  Pattern 2

Pattern 1  Pattern 2
Extensions

• This work will be extended to study:
  – other non-stationary demand patterns.
  – part priorities
  – product mix changes.
  – buffer sizing for workstations.
  – effect of different scheduling disciplines.
  – capacity/workstation availability conditions.
  – More memory efficient (single stage) WS level closed approximations.
Conclusions

• All hope is not lost, we can tractably do rough cut estimation
• More work is needed
Thanks for Listening

Questions?