Analysis of Flexible Serial Lines with Setups

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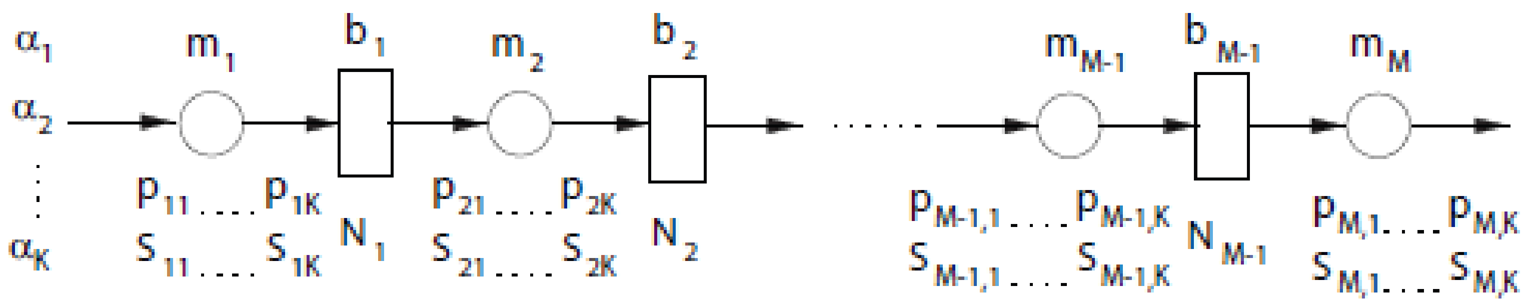
June 2, 2015

Outline

- System Modeling
- Single Machine Line
- Two-Machine Line
- Long Serial Line
- Bottleneck Analysis
- Case study
- Summary

System Modeling

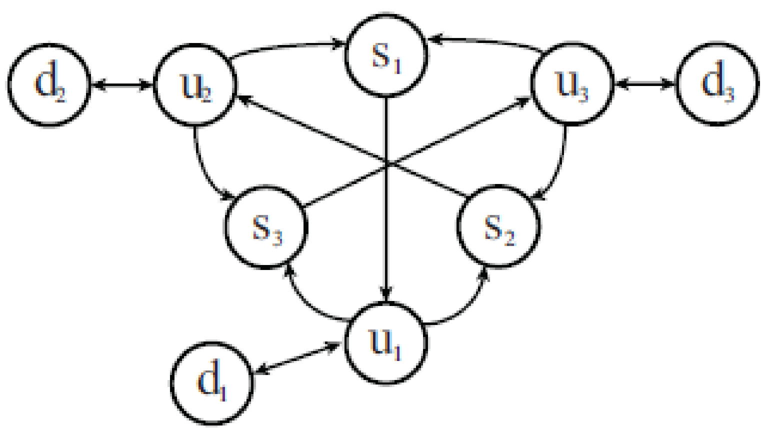
- Serial Production Line with M machines and K products



- Incoming sequence
  - product type: discrete distribution with probability α_j to be type j
  - $\sum_{j=1}^{K} \alpha_j = 1$
- Bernoulli reliability model
  - Parameters p_{ij}, for machine i product j
- Bernoulli setup model
  - Setup success with probability s_{ij}

Single Machine

- A single machine with $K = 3$ products.
- State transition highlights
  - Failed machine must return to the same type up state.
  - Product changeover must go through setup.
  - Self loops are ignored.



Transition diagram

$$\pi_{u_{1j}} = \frac{\alpha_j}{1 + \sum_{l=1}^{K} \frac{\alpha_l(1-\alpha_l)}{s_{1l}} + \alpha_l^2\left(\frac{1}{p_{1l}} - 1\right)}$$

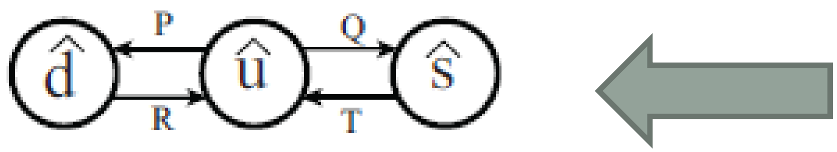
$$\pi_{s_{1j}} = \frac{\frac{\alpha_j(1-\alpha_j)}{s_{1j}}}{1 + \sum_{l=1}^{K} \frac{\alpha_l(1-\alpha_l)}{s_{1l}} + \alpha_l^2\left(\frac{1}{p_{1l}} - 1\right)}$$

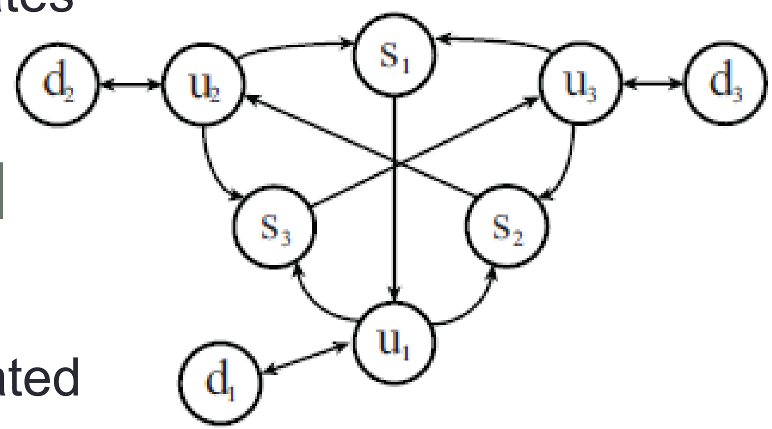
$$\pi_{d_{1j}} = \frac{\alpha_j^2\left(\frac{1}{p_{1j}} - 1\right)}{1 + \sum_{l=1}^{K} \frac{\alpha_l(1-\alpha_l)}{s_{1l}} + \alpha_l^2\left(\frac{1}{p_{1l}} - 1\right)}$$

Steady state distribution

Single Machine State Aggregation

- Scale up the analytical method for long line
  - Group into down, up and setup states





  - Transition parameters are aggregated

Breaks down

Still make product *j*

$$P = \frac{\sum_j (1-p_{1j})\alpha_j \pi_{u_{1j}}}{\sum_j \pi_{u_{1j}}}$$

From an up state

$$R = \frac{\sum_j p_{1j}\pi_{d_{1j}}}{\sum_j \pi_{d_{1j}}}$$

$$Q = \frac{\sum_j (1-\alpha_j)\pi_{u_{1j}}}{\sum_j \pi_{u_{1j}}}$$

$$T = \frac{\sum_j s_{ij}\pi_{s_{1j}}}{\sum_j \pi_{s_{1j}}}$$

Single Machine State Aggregation

- Parameter aggregation

$$P = \frac{\sum_j (1-p_{1j})\alpha_j \pi_{u_{1j}}}{\sum_j \pi_{u_{1j}}} = \sum_j (1-p_{1j})\alpha_j^2,$$
$$R = \frac{\sum_j p_{1j}\pi_{d_{1j}}}{\sum_j \pi_{d_{1j}}} = \frac{P}{\sum_j \alpha_j^2 (1/p_{1j} - 1)},$$
$$Q = \frac{\sum_j (1-\alpha_j)\pi_{u_{1j}}}{\sum_j \pi_{u_{1j}}} = \sum_j (1-\alpha_j)\alpha_j,$$
$$T = \frac{\sum_j s_{ij}\pi_{s_{1j}}}{\sum_j \pi_{s_{1j}}} = \frac{Q}{\sum_j (1-\alpha_j)\alpha_j / s_{ij}}.$$

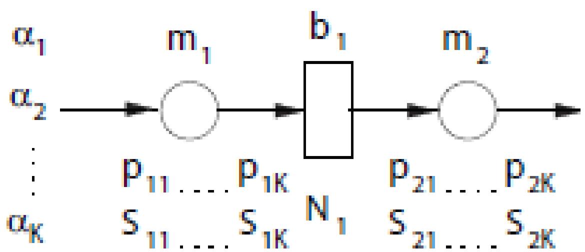
- Steady states are computed and matched with original machine.

$$\pi_{\hat{u}} = \frac{1}{P/R + 1 + Q/T}, \qquad \pi_{\hat{s}} = \frac{Q/T}{P/R + 1 + Q/T}, \qquad \pi_{\hat{d}} = \frac{P/R}{P/R + 1 + Q/T}.$$

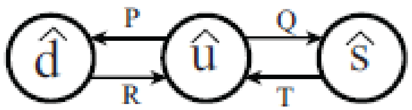
- Throughput for product type *j*
  - $\pi_{u_{1j}} = \alpha_j \pi_{\hat{u}}$

Two-Machine Line

- Full state space size: $9K^2(N^K + 1)$
- Aggregated state space size: $9(N + 1)$



For each of the machines



- Performance evaluation (subscripts P, R, Q, T are machine indexes for m_1 and m_2):

$$PR = \Phi_p(P_1, P_2, R_1, R_2, Q_1, Q_2, T_1, T_2, N) = \pi_{N,uu} + \sum_{k=0}^{N-1}(\pi_{k,us} + \pi_{k,ud} + \pi_{k,uu})$$

$$= \sum_{k=1}^{N}(\pi_{k,su} + \pi_{k,du} + \pi_{k,uu}),$$

$$BL = \Phi_b(P_1, P_2, R_1, R_2, Q_1, Q_2, T_1, T_2, N) = \pi_{N,us} + \pi_{N,ud},$$

$$ST = \Phi_s(P_1, P_2, R_1, R_2, Q_1, Q_2, T_1, T_2, N) = \pi_{0,uu} + \pi_{0,su} + \pi_{0,du},$$

$$WIP = \Phi_w(P_1, P_2, R_1, R_2, Q_1, Q_2, T_1, T_2, N) = \sum_k \sum_{m_1} \sum_{m_2} k \cdot \pi_{k,m_1 m_2}.$$

Long Serial Line

- Use blockage and starvation information to adjust production failure parameters P_i and R_i.

Procedure 1

$$
\begin{aligned}
bl_i^b(n+1) &= \Phi_b(P_i^f(n), P_{i+1}^b(n+1), R_i^f(n), R_{i+1}^b(n+1), Q_i, Q_{i+1}, T_i, T_{i+1}, N_i),\\
st_i^f(n+1) &= \Phi_s(P_{i-1}^f(n), P_i^b(n+1), R_{i-1}^f(n), R_i^b(n+1), Q_{i-1}, Q_i, T_{i-1}, T_i, N_{i-1}),\\
P_i^b(n+1) &= P_i + R_i bl_i^b(n+1), \quad R_i^b(n+1) = R_i(1 - bl_i^b(n+1)), \qquad i = 1, \ldots, M,\\
P_i^f(n+1) &= P_i + R_i st_i^f(n+1), \quad R_i^f(n+1) = R_i(1 - st_i^f(n+1)), \qquad i = 1, \ldots, M
\end{aligned}
$$

with initial conditions

$$P_i^f(0) = P_i, \quad R_i^f(0) = R_i, \qquad i = 1, \ldots, M$$

and boundary conditions

$$P_1^f(n) = P_1, \quad P_M^b(n) = P_M, \quad R_1^f(n) = R_1, \quad R_M^b(n) = R_M, \quad n = 0, 1, 2, \ldots.$$

- The algorithm converges.

Accuracy Evaluation

- 10,000 production lines are randomly generated.

$$
\begin{aligned}
& M \in \{2,\dots,10\}, \qquad K \in \{2,\dots,5\}, \\
& p_{ij} \in (0.7, 0.99), \quad i = 1,\dots,M, j = 1,\dots,K, \\
& s_{ij} \in (0.5, 0.99), \quad i = 1,\dots,M, j = 1,\dots,K, \\
& N_i \in \{5,6,\dots,15\}, \quad i = 1,\dots,M-1, \\
& \alpha_j \in (0,1), \quad j = 1,\dots,K, \quad s.t. \sum_{j=1}^{K} \alpha_j = 1.
\end{aligned}
$$

- Computation results are compared with simulation results.
  - Production rate: avg err 2.90%
  - WIP: avg err 9.40%
  - BL,ST: avg err 0.02

$$
\begin{aligned}
\delta_{PR} &= \frac{|PR^{sim} - PR^{model}|}{PR^{sim}} \cdot 100\%, \\
\delta_{WIP_i} &= \frac{|WIP_i^{sim} - WIP_i^{model}|}{N_i} \cdot 100\%, \\
\delta_{BL_i} &= |BL_i^{sim} - BL_i^{model}|, \\
\delta_{ST_i} &= |ST_i^{sim} - ST_i^{model}|,
\end{aligned}
$$

Bottleneck Product Type in Setup Time

- Focus on one machine, find the most severe setup bottleneck type.

DEFINITION 1. Under assumption 1-9, product type j on machine m_i is the setup bottleneck product type if

$$\left|\frac{\partial PR}{\partial T_{s_{ij}}}\right| > \left|\frac{\partial PR}{\partial T_{s_{il}}}\right|, \qquad \forall l \neq j, l, j \in \{1, \ldots, K\},$$

where $T_{s_{ij}} = \frac{1}{s_{ij}}$ is the setup time.

- Recall

$$Q = \frac{\sum_j (1-\alpha_j)\pi_{u_{1j}}}{\sum_j \pi_{u_{1j}}} = \sum_j (1-\alpha_j)\alpha_j,$$

$$T = \frac{\sum_j s_{ij}\pi_{s_{1j}}}{\sum_j \pi_{s_{1j}}} = \frac{Q}{\sum_j (1-\alpha_j)\alpha_j/s_{ij}}.$$

$$\pi_{\hat{u}} = \frac{1}{P/R + 1 + Q/T}$$

- Indicator

PROPOSITION 3. *Under assumptions 1-9, for machine m_i, the setup bottleneck product type j is the one with the largest value of $(1-\alpha_j)\alpha_j$, or equivalently, the smallest value of $|\alpha_j - 0.5|$.*

Selectivity of Setup and Failure Improvement

- On a given machine
  - focus on setup time reduction or downtime reduction?

DEFINITION 2. Under assumptions 1-9, for machine m_i, setup time reduction has higher selectivity in continuous improvement if

$$\sum_{j=1}^{K} \left| \frac{\partial PR}{\partial T_{s_{ij}}} \right| > \sum_{j=1}^{K} \left| \frac{\partial PR}{\partial T_{d_{ij}}} \right|, \qquad \forall i \in \{1, \ldots, M\}.$$

- Recall

$$\pi_{u_{1j}} = \frac{\alpha_j}{1 + \sum_{l=1}^{K} \frac{\alpha_l(1-\alpha_l)}{s_{1l}} + \alpha_l^2 \left(\frac{1}{p_{1l}} - 1 \right)}$$

- Indicator

PROPOSITION 4. *Under assumptions 1-9 with $M = 1$, setup time reduction has higher selectivity if and only if* $\sum_j \alpha_j^2 < 0.5$.

- Accuracy for long line ($M \geq 2$)
  - 84.54% out of 10,000 cases, it finds the right selectivity.
  - In the failed cases, derivatives are small.
  - 92.64%: all machines in a product line have the same selectivity.

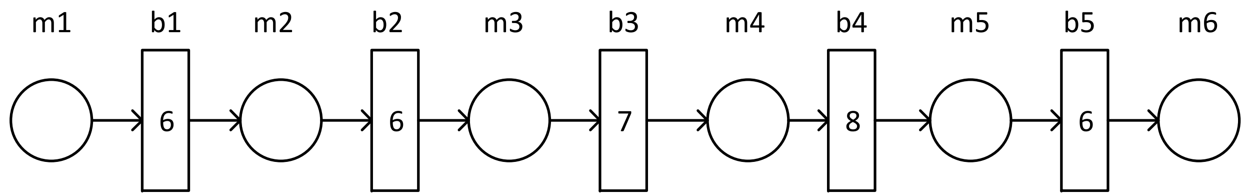
Showcase and Effectiveness

	m1	m2	m3	m4	m5	m6
p_{i1}	0.9708	0.9834	0.8473	0.8468	0.8013	0.9700
p_{i2}	0.8108	0.7334	0.9341	0.8169	0.7725	0.8212
p_{i3}	0.7289	0.7396	0.9826	0.9868	0.8726	0.7179
s_{i1}	0.6939	0.7413	0.9285	0.6062	0.6172	0.6676
s_{i2}	0.8596	0.8927	0.8591	0.7804	0.8188	0.7185
s_{i3}	0.8979	0.6756	0.8747	0.6734	0.7474	0.8502

α_1 0.4055

α_2 0.1303

α_3 0.4642



	m1	m2	m3	m4	m5	m6
$\Delta PR/\Delta T_{s_{i1}}$	0.0000	0.0001	0.0003	0.0207	0.0666	0.0185
$\Delta PR/\Delta T_{s_{i2}}$	0.0000	0.0000	0.0001	0.0098	0.0314	0.0086
$\Delta PR/\Delta T_{s_{i3}}$	0.0000	0.0001	0.0003	0.0214	0.0688	0.0191
$\sum_j \Delta PR/\Delta T_{s_{ij}}$	0.0000	0.0001	0.0008	0.0519	0.1667	0.0462
$\sum_j \Delta PR/\Delta T_{d_{ij}}$	0.0000	0.0001	0.0003	0.0285	0.1067	0.0317

α_3 most close to 0.5, product 3 is bottleneck

$\sum \alpha_j^2 < 0.5$ Setup time reduction is more effective

Bottleneck Setup Machine

- On a serial line
  - Which machine is most impeding in terms of setup time?

DEFINITION 3. Machine m_i is the joint setup bottleneck machine if

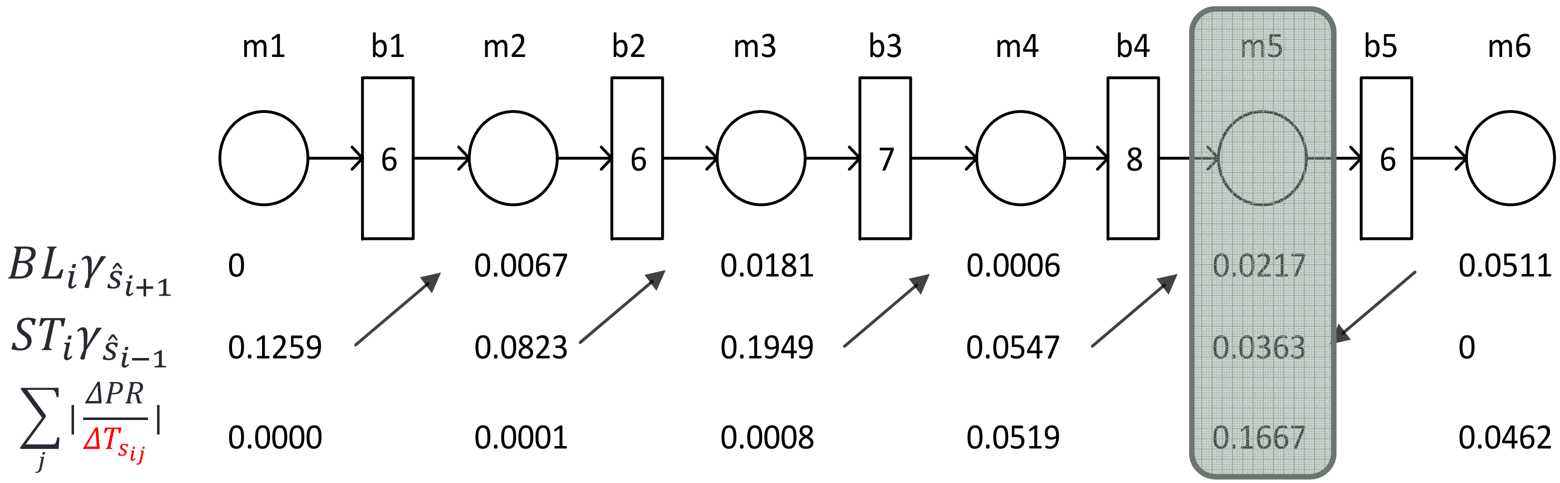
$$\sum_{j=1}^{K} \left|\frac{\partial PR}{\partial T_{s_{ij}}}\right| > \sum_{j=1}^{K} \left|\frac{\partial PR}{\partial T_{s_{lj}}}\right|, \quad \forall i \neq l \in \{1, \ldots, M\}.$$

  - Indicator is based on the adjusted starvation and blockage.
    - Use downstream setup percentage ($\gamma_{\hat{s}_{i+1}}$) in idle time to adjust blockage/starvation
    - Use upstream setup percentage ($\gamma_{\hat{s}_{i-1}}$) in idle time to adjust blockage/starvation
  - Arrow assignment rule
    - $BL_i \gamma_{\hat{s}_{i+1}} > ST_{i+1} \gamma_{\hat{s}_i}$, assign the arrow from m_i to m_{i+1}.
    - $BL_i \gamma_{\hat{s}_{i+1}} < ST_{i+1} \gamma_{\hat{s}_i}$, assign the arrow from m_{i+1} to m_i.
  - In case of multiple bottleneck, severity index is used.

$$S_i = |ST_{i+1}\gamma_{\hat{s}_i} - BL_i\gamma_{\hat{s}_{i+1}}| + |BL_{i-1}\gamma_{\hat{s}_i} - ST_i\gamma_{\hat{s}_{i-1}}|, i = 2, \ldots, M-1,$$
$$S_1 = |ST_2\gamma_{\hat{s}_1} - BL_1\gamma_{\hat{s}_2}|,$$
$$S_M = |BL_{M-1}\gamma_{\hat{s}_M} - ST_M\gamma_{\hat{s}_{M-1}}|.$$

Showcase and Effectiveness

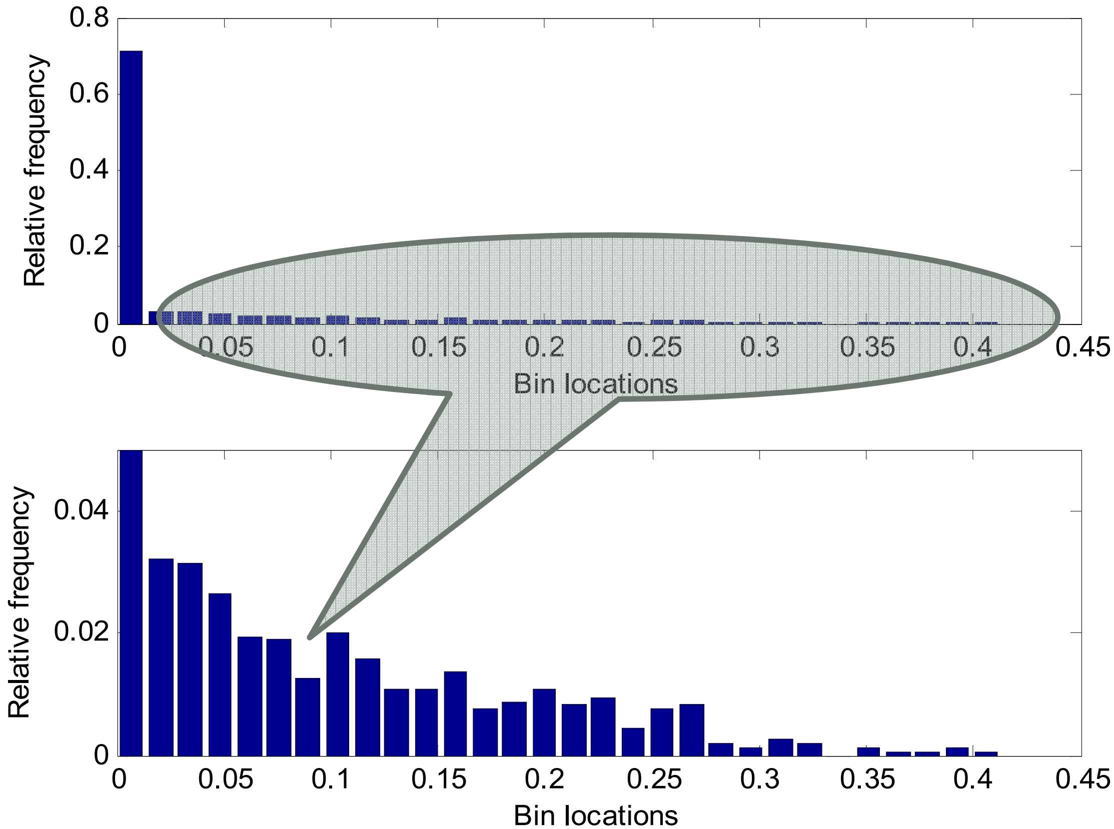
- A randomly generated serial line



- Accuracy evaluation:
  - 2000 lines generated
  - 80%: Single BN cases
  - 20%: Multiple BN cases. 70%: true BN machine is in candidate set.

Effectiveness

- Simulation comparison:
  - Single BN cases: True and identified BNs, difference of derivatives

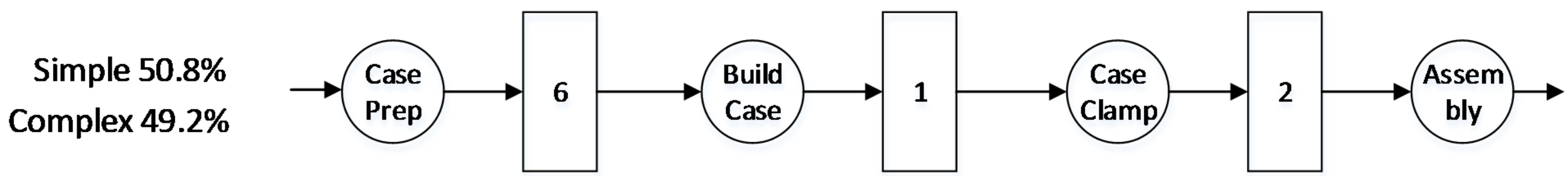


  - Multiple BN cases

    Similar histogram pattern. Avg difference using severity index: 0.06

A Case Study

- Techline furniture assembly line.
  - Build main body in a serial line, assemble with drawers and doors at the end. Manual operations.
  - Feeding and shipping inventory large enough.
  - Simple and complex product groups.
  - Setup used for workers to read instructions and prepare tools.

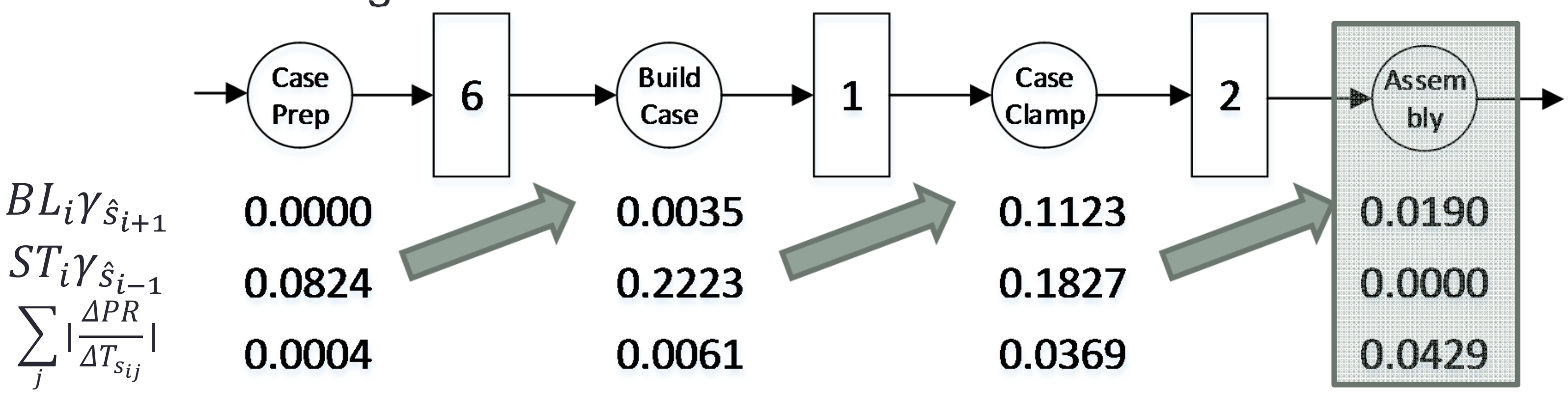


	Case Prep	Build Case	Case Clamp	Assembly
p_{i1}	0.902	0.397	0.999	0.434
p_{i2}	0.535	0.397	0.999	0.277
s_{i1}	0.630	0.958	0.958	0.495
s_{i2}	0.495	0.958	0.958	0.407

- Throughput 205 (from production count) vs 218 (from model)
  - Model validated

Bottleneck Analysis

- Bottleneck indicators
  - Setup bottleneck product
    - $|0.5-\alpha_1| = |0.5-\alpha_2|$, same effectiveness, but reduce setup time for complex product is more practical.
  - Bottleneck Machine
    - Arrow assignment finds the true bottleneck machine



- Reduce 20% setup time on simple and complex assembly station
  - The throughput increase 3.21% (7 products).

Summary

- Effective tools to evaluate flexible production systems with setups are developed.
- Bottlenecks setup machine and bottleneck product are studied.
- Application in a furniture assembly line is carried out.

- Future work
  - Other reliability models (exponential, general)
  - Product sequencing and production control
  - Transient analysis

Call for Papers

- **Special Issue:**
  - ***Continuous Improvement in Manufacturing and Service Systems***
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- **Important Dates:**
  - **Manuscript submission: June 30, 2015**
  - Tentative publication date: September 2016