Stochastic Inventory Routing Problem with Transshipment Recourse Action

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Vendor Managed Inventory (VMI) systems seems to be at the core of most global supply chains.

**Concept:** The replenishment & the distribution making process is centralized at supplier level. Supplier acts as central decision maker. This policy leads to an overall reduction of logistic cost.

**Advantage:** More efficient resource utilization. Often described as a win – win situation.

**Inventory Routing Problem (IRP)** constitutes the backbone of the VMI systems.

Decision to be taken are:

1. **When** to deliver to each customer
2. **How much** to deliver to each customer each time it is served
3. **How to route** the vehicles so as to minimize the total cost.
Motivation

• Need of major electronics multinationals with production both in Asia and Europe and various warehouses throughout their global supply chain management for more efficient resource utilization.
• Need to adjust their operations to meet the requirements of their clients.
• Need to account the uncertainty of demand.
The problem was first introduced by Bell et al. (1983) and Federgruen & Zipkin (1984).

To the best of our knowledge there are two seminal papers regarding literature review on the IRP Andersson et al. (2010) related to business models and classification of problems and Coelho et al. (2014) related to methods and algorithms.

On the other hand Geisen, Mahmassani and Jaillet (2009) and Rabah and Mahmassani (2002) provide an excellent reference for applications of VMI policies with stochastic demand.

Bertazzi, Paletta and Speranza (2002) introduced a practical VMI policy the deterministic Order – Up – to level policy.

Arhetti et al. (2007) developed the first exact method based on the OU-Policy.

Coelho & Laporte (2012) introduce the transshipment cost within IRP and developed an exact method as well as an ALNS metaheuristic for large scale instances.
IRP under uncertainty

- Campbell et al. (1998) set the basis for the rolling horizon framework.
- Kleywegt et al. (2002, 2004) formulated the stochastic IRP as a Markov Decision Process (MDP) over an infinite horizon.
- Solyali et al. (2012) proposed two mixed integer programming formulations of the robust version of the problem, which produce policies leading to feasible solution and optimal cost for any realization of the demand.
- Bertazzi et al. (2013) as well as Coelho (2012) rely on a dynamic programming formulation that allows the design of a hybrid rollout formulation aiming to find good quality solution.
• Introduce a stochastic programming model for the IRP and propose an L – Shaped algorithm that efficiently solves the SIRP using transshipment as recourse action.

• Introduce new valid inequalities for the first stage decision process which accounting forthcoming time period demand to determine the delivered quantities.
First Stage Decision Process: Inventory Routing Problem

\[
\min c^T x \\
\text{Subject to: } Ax = b, \\
x \geq 0
\]

Second Stage Decision Process: Transshipment Recourse Actions

\[
\min c^T x + \sum_{\omega \in \Omega} p(\omega) d^T \omega y_{\omega} \\
\text{Subject to: } T_{\omega} x + W_{\omega} y = h_{\omega}, \\
x > 0, y_{\omega} > 0
\]
Methodology

Algorithm: The L – shaped algorithm for stochastic inventory with transshipment

1: for $t=0$
2:   compute the IRP plan by solving the first stage decision process
   (Obj. Function: (12) s.t. (13) - (34))
3:   for each scenario $\omega \in \Omega$ do
4:       compute the dual multiplier and produce the feasibility cut
5:       add feasibility cut to the master problem
6:   endfor
7:   for each scenario $\omega \in \Omega$ solve the sub problem
   (Obj. Function: (35) s.t. (36) - (38))
8:       compute optimal value and optimal dual value
9:   endfor
10: compute UB and execute convergence test
11: If $\frac{UB-LB}{UB} \leq 0.001$ then
12:   Stop required accuracy achieved
13: Return $x^*$
14: Endif
15: solve the master problem Objective function (43)
16: add Optimality cuts
17: Compute optimal values and LB
18: return to 11
19: endfor

1st STEP: Solve first stage MIP - IRP

2nd STEP: for each scenario Solve 2nd stage MIP

3rd STEP: Perform convergence test

4th STEP: Create optimality cut

While desired accuracy not met

Add optimality cut

Determine Quantities & Routes

Determine expected transshipment cost

START

END

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\[
\text{minimize } \sum_{t \in T} \sum_{i \in V} h_i I_i^t + \sum_{t \in T} \sum_{i \in V} \sum_{i \neq j} c_{ij} x_{i,j}^t
\]
with recourse action of transshipment

\[
\min \sum_{t \in T} \sum_{i \in V} h_i t_i^* + \sum_{t \in T} \sum_{i \in V} \sum_{j \in V} c_{ij} x_{ij}^t + E^* \left\{ \sum_{t \in T} \sum_{j \in V} \alpha * c_{ij} w_{ij}^t(\omega) \right\}
\]
Deterministic Equivalent

\[ \min \sum_{t \in T} \sum_{i \in V} h_i^t + \sum_{t \in T} \sum_{i \in V} \sum_{j \neq i} c_{ij} x_{ij}^t + \sum_{i=1}^{T-1} p_i \left\{ \sum_{t \in T} \sum_{j \in B \cap V} \alpha + c_{ij} y_{ij}^t (\omega^t) \right\} \]

\[ I_i^t \geq 0 \quad \forall t \in T \]
\[ I_i^t \geq I_i^{t-1} + r^t - \sum_{i \in V'} q_i^t \quad \forall t \in T \]
\[ I_i^t \geq 0 \quad \forall t \in T, \forall i \in V' \]
\[ I_i^t \geq I_i^{t-1} + q_i^t - d_i^t \quad \forall t \in T, \forall i \in V' \]
\[ \sum_{t \in T} q_i^t \leq \sum_{j \in V'} d_i^j, \forall i \in V', \forall t \in T \]
\[ \sum_{j=1}^{t} q_i^j \leq r_i^0 + \sum_{j=1}^{t} r_i^j, \forall t \in T, \forall i \in V' \]
\[ I_i^t \leq C_i \quad \forall t \in T, \forall i \in V' \]
\[ \sum_{i \in V'} q_i^t \leq C \quad \forall t \in T \]

\[ I_i^t = I_i^{t-1} - \sum_{k \in E, k \neq i} w_{ik}^t (\omega) + \sum_{k \in E, k \neq i} w_{ki}^t (\omega) \geq 0, \forall t \in T, \forall i \in V \]
\[ I_i^t = I_i^{t-1} - \sum_{k \in E, k \neq i} w_{ik}^t (\omega) + \sum_{k \in A, k \neq i} w_{ki}^t (\omega) + q_i^t - d_i^t (\omega) \quad \forall t \in T, \forall i \in V' \]
\[ I_i^t = I_i^{t-1} + r^t - \sum_{i \in V' \setminus E} w_{ii}^t (\omega) - \sum_{i \in V' \setminus E} q_i^t \quad \forall t \in T \]
\[ \sum_{t \in T} \sum_{j \in V} x_{ij}^t \leq H \]
\[ \sum_{j \in V'} x_{ij}^t \leq y_i^t \quad \forall t \in T \]
\[ \sum_{j \in V'} x_{ij}^t + \sum_{j \in V'} x_{ij}^t = 2y_i^t \quad \forall t \in T, \forall i \in V' \]
\[ x_{ij}^t \leq y_i^t \quad \forall t \in T, \forall i, j \in V' \]
\[ C (1 - x_{ij}^t) + u_{ij}^t \geq u_{ij}^t + q_{ij}^t \quad \forall i, j \in V', \forall t \in T, \forall i \neq j, t \in T \]
\[ q_i^t \leq u_i^t \quad \forall i, j \in V', t \in T \]
\[ u_i^t \leq y_i^t * C \quad \forall i \in V', t \in T \]
\[ y_i^t \leq y_i^t \quad \forall t \in T, \forall i \in V' \]
\[ q_i^t, u_i^t \geq 0 \quad \forall i \in V', t \in T \]
\[ x_{ij}^t \in \{0,1\} \quad \forall i, j \in V', \forall t \in T \]
\[ y_i^t \in \{0,1\} \quad \forall i \in V, t \in T \]
Algorithm was coded in ILOG Optimization Studio CPLEX 12.4.
Benchmark instances of Arhetti et al. (2007) were used to evaluate the proposed valid inequalities.
• Transshipment significantly improves the overall performance of vendor managed inventory supply chain.

• Relaxation of the Order – Up to level policy in coherence to the decision of accounting forthcoming demand to determine the quantity of shipments demonstrate savings of 15% on an average.
Evaluation of Transshipment costs

Results

Total Costs (First Stage+Second Stage)

Cost of Recourse Action

Final IRP Solution

First Stage Cost

Alternative Scenarios of recourse costs
• Nowadays of unstable global economic conditions the demand of products become highly uncertain in many business areas.

• **Sustainability** of business depend on the ability to handle market uncertainties.

• Research should focus on development of models and methods that fit the industries needs of **robust** flexible plans to handle the uncertainties.
Thank you for your attention

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