Abstract: In a reverse supply chain, the location where returned items are sorted may have substantial impact on the profitability of the chain. The present paper contributes to the quantification of the trade-offs related with this decision by examining and comparing two alternative options which can be considered as the two extremes with respect to the physical distance and the time delay between sorting and recovery (remanufacturing) procedures. Specifically, we analyze a simple reverse supply chain consisting of a collection site and a recovery site in a single-period setting. The system faces uncertainty due to two factors: uncertainty in the quality of returns and uncertainty in the demand for recovered products. We provide analytical and computational results that are useful in determining when a centralized system (sorting at the remanufacturing facility) is preferable to a decentralized one (sorting at the collection site).

Keywords: reverse logistics, random yield, single-period, remanufacturing

1. Introduction
Among the issues arising during the design of reverse supply networks an important one is the choice of the location where the sorting procedures will take place. By the term sorting procedures we refer to the sequence of actions that lead to the classification of the returned units on the basis of their quality state in order to assign them to a proper reuse procedure. The two extreme possibilities regarding the location of the sorting procedures are: a) the centralized system where all sorting procedures take place at the recovery sites; and b) the decentralized system, where all sorting procedures are carried out at the collection sites where consumers return used products. Since reverse supply networks typically consist of numerous collection sites and a relatively small number of recovery facilities, the main trade-off in comparing these two extreme solutions is between higher transportation costs and delayed information flow for the centralized network and higher operating/sorting costs for the decentralized network.

Fleischmann (2001) was among the first to deal with the problem of selection between centralized versus decentralized sorting procedures. By modeling the returned items volume as a “continuous geographic density function” and introducing the system parameters as functions of the geographical coordinates he derives expressions for the maximum area size that is optimal to be served by a centralized sorting facility. Returns located outside that area have to be sorted using local sorting facilities.

Although we deal with a problem similar to Fleischmann (2001) our approach is quiet different since we assume that the returns are being collected at a specific location, the collection site. The simple supply chain we study also includes a remanufacturing facility which faces stochastic demand for remanufactured items. The quality (remanufacturability) of the returned units can be revealed only after sorting, which can be carried out either at the collection site or at the remanufacturing facility. The objective of this paper is to provide the means (models) for analyzing both options, so as to be in a position to determine the most profitable one.

The models examined here share common features with previous work in the context of forward supply chains with uncertainty in the supply side, in terms of quality or quantity (stochastic yield). For example, Shih (1980) studies a production system supplied by a unique supplier with the objective of
minimizing total expected costs. The supply yield and the demand for finished products are random variables, which follow general distributions. Shih (1980) assumes that after the order arrival and the realization of the yield, the full amount of useable products is processed in order to meet the demand. The centralized model studied here differs from Shih (1980) in that the amount of useable (remanufacturable) products to be processed is a decision variable.

Bassok and Akella (1991) modify the model of Shih (1980) by including initial inventory and by allowing the yield distribution to depend on the quantity ordered. Additionally, they study the same problem for the multiple products case. Henig and Levin (1992) are concerned with simultaneous determination of the quantity to procure from a single supplier and the quantity to deliver to customers subject to production capacity constraints. They also discuss issues of dominance of a single supplier among several candidates. Parlar and Wang (1993) study and optimize a system that includes two suppliers with independent stochastic yield distributions and stochastic demand. Like Shih (1980), they assume that after the order arrival and the realization of the yield, the full amount of useable products is processed to meet the demand. A complete literature review of single-period lot sizing problems with stochastic yield can be found in Yano and Lee (1995).

The most relevant paper that deals with a single-period problem similar to ours in a reverse logistics context is that of Ferrer (2003), which examines a remanufacturing system with one collection site. The yield of the returned products is a random variable but demand is known. The objective is to study the impact of the timing of the information regarding the quality of the returns on the profitability of reuse activities. Our system is more complex than Ferrer’s in the sense that the demand for remanufactured products is a random variable. On the other hand, the model in Ferrer (2003) is richer in that it includes an option to satisfy the demand for remanufactured products with new ones. The model studied by Ferrer (2003) can be considered as a modification of Henig and Levin (1992) in the reverse supply chain context, while ours can be viewed as a modification of that of Shih (1980).

The next section provides a detailed description of the problem and introduces the notation. Sections 3 and 4 present the expected profit functions for the centralized and the decentralized system respectively and explain how to obtain the optimal solutions. Section 5 contains a numerical investigation and finally Section 6 concludes the paper with a summary and ideas for future research.

2. Problem description

We consider a reverse supply chain consisting of a collection site (CS), where ample quantity of used products is collected and a remanufacturing facility (R). Used products can be in one of two possible quality states: i. remanufacturable and ii. non-remanufacturable. The proportion of remanufacturables at CS, q, is treated as continuous random variable following a distribution with known probability density function g(q) and distribution function G(q). R faces random demand for remanufactured products which may be satisfied by remanufacturing (in advance) a certain quantity of the returned units. The production lot size, s_r, can be determined optimally, taking into account the relevant costs and revenues as well as the yield and demand distributions characteristics. The remanufacturable units can be identified among the returns through an error-free sorting procedure that may take place either at CS or at R.

When sorting takes place at CS (local sorting) the production lot size at R coincides with the quantity ordered from CS (procurement lot size, s_t). Sorting at CS stops when exactly s_t remanufacturable units have been identified. Sorting incurs a cost, c_sd, per unit inspected. After sorting, the s_r=st remanufacturable units are transported to R at a cost of c_t per unit, while non-remanufacturable units are disposed of at no cost.

When sorting takes place at R (central sorting), the quantity to be transported from CS to R, s_r, is a decision variable. The transportation cost per unit is the same as in the decentralized system but in this case an unknown number of non-remanufacturable units are also transported to R. The inspection cost per unit at R is c_sc. After sorting, the exact number of available remanufacturable units, denoted by s, becomes known. The units that will not be remanufactured either because they are not
remanufacturable or because there are more remanufacturable units available than $s_r$, are disposed of at a cost of $c_d$ per unit.

The demand for remanufactured products follows a distribution with probability density function $f(.)$ and distribution function $F(.)$. The remanufactured units are sold at a price of $v > c_r$ per unit. If the demand exceeds $s_r$, a shortage cost $b$ is charged per unit short. In the opposite case, the excess quantity is treated exactly like non-remanufacturable items. The notation used throughout the paper is summarized in Table 1.

### Table 1: Notation

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_t$:</td>
<td>quantity transported from CS to R (procurement lot size)</td>
</tr>
<tr>
<td>$s_r$:</td>
<td>quantity to remanufacture (production lot size)</td>
</tr>
<tr>
<td>$s$:</td>
<td>quantity of available remanufacturable units at R</td>
</tr>
<tr>
<td>$c_{sd}$:</td>
<td>sorting cost per unit (local)</td>
</tr>
<tr>
<td>$c_{sc}$:</td>
<td>sorting cost per unit (central)</td>
</tr>
<tr>
<td>$c_t$:</td>
<td>transportation cost from CS to R per unit</td>
</tr>
<tr>
<td>$c_d$:</td>
<td>disposal cost at R per unit</td>
</tr>
<tr>
<td>$c_r$:</td>
<td>remanufacturing cost per unit</td>
</tr>
<tr>
<td>$v$:</td>
<td>sales revenue per unit</td>
</tr>
<tr>
<td>$b$:</td>
<td>shortage cost per unit</td>
</tr>
<tr>
<td>$q$:</td>
<td>fraction of remanufacturables (random variable)</td>
</tr>
<tr>
<td>$s$:</td>
<td>quantity of available remanufacturable units at R</td>
</tr>
<tr>
<td>$q$:</td>
<td>fraction of remanufacturables (random variable)</td>
</tr>
<tr>
<td>$D$:</td>
<td>demand for remanufactured units (random variable)</td>
</tr>
<tr>
<td>$g(.)$:</td>
<td>probability density function of $q$</td>
</tr>
<tr>
<td>$f(.)$:</td>
<td>probability density function of $D$</td>
</tr>
<tr>
<td>$G(.)$:</td>
<td>distribution function of $q$</td>
</tr>
<tr>
<td>$F(.)$:</td>
<td>distribution function of $D$</td>
</tr>
</tbody>
</table>

### 3. Central sorting

The sorting procedure at R separates two distinct decisions which concern the determination of the procurement lot size $s_t$ (transportation quantity) and the production (remanufacturing) lot size $s_r$. We therefore approach the model formulation and optimization in two stages, moving backwards in decision time. We start with the second decision stage, i.e., with the formulation of the expected profit for a given quantity $s$ of available remanufacturable items so as to determine the optimal $s_r$. Then, we proceed to the first decision stage where we formulate the total expected profit function that leads to the determination of the optimal $s_t$.

#### 3.1 The optimal production lot size $s_r$ (as a function of $s$)

The amount $s$ of units available for remanufacturing at R after the sorting is completed is a realization of the random variable $s_q$. For a given value of $s$, the only remaining source of uncertainty is the demand ($D$). The decision that needs to be made is the quantity $s_r \leq s$ of items that will undergo the remanufacturing process. The expected profit as a function of $s_r$ can be written as:

\[
E[TP(s_r)] = \left[ v \int_0^{s_r} xf(x)dx \right] + \int_0^{s_r} s_r f(x)dx \] (sales revenue)

- $b \int_{s_r}^\infty (x - s_r)f(x)dx$ (shortage cost)

- $c_d \int_0^{s_r} (s_r - x)f(x)dx$ (disposal cost of excess remanufactured)

- $c_r s_r$ (remanufacturing cost)

- $c_d (s - s_r)$ (disposal cost of excess remanufacturables)

After some mathematical manipulation the expected profit function takes the form:

\[
E[TP(s_r)] = (v + b + c_d) \int_0^{s_r} f(x)dx + (v + b + c_d - c_r)s_r - c_d s_r
\] (1)

The above expected profit function must be optimized subject to the constraint
In order to find the optimal solution we first solve the unconstrained problem and we check if (2) is satisfied. Taking the first derivative of \(E[TP(s_r)]\) with respect to \(s_r\) and setting it equal to zero, yields:

\[
F(s_r) = (v + b + c_d - c_r)/(v + b + c_d).
\] (3)

The second derivative of \(E[TP(s_r)]\) with respect to \(s_r\) is non-positive, which confirms that the value of \(s_r\) that satisfies (3) is the global maximum of the unconstrained problem. Since we have assumed that \(v, b, c_d\) and \(c_r\) are positive, and \(v > c_r\) the r.h.s. of (3) takes values in the interval \((0, 1)\). Therefore, there will always be a non-negative value of \(s_r\), say \(s^*_r\), for which (3) holds.

If \(s^*_r\) is not greater than \(s\), then the inclusion of constraint (2) does not alter the optimal solution. On the other hand, if \(s^*_r > s\), then \(s^*_r\) is not a feasible solution of the constrained problem. In that case, since \(dE[TP(s_r)]/ds_r \geq 0\) for every \(s_r\) in \([0,s]\), the optimal value of \(s_r\) is \(s\). Thus, knowing the number of available remanufacturable units, \(s\), and the value of \(s^*_r\), the optimal policy for that stage is the following:

- if \(s > s^*_r\), then dispose of \(s - s^*_r\) units and remanufacture the remaining \(s^*_r\) units;
- if \(s \leq s^*_r\), then remanufacture all the available \(s\) units.

3.2 The optimal procurement lot size \(s_t\)

For mathematical convenience we examine separately the two areas of possible \(s_t\) values, namely \(s_t < s^*_t\) and \(s_t \geq s^*_t\). Whenever the procurement lot size \(s_t\) is smaller than \(s^*_t\) (\(s_t < s^*_t\)) it is clear that \(s \leq s^*_t\) (since \(s = s_t q \leq s_t\)) and consequently the optimal production lot size coincides with \(s\). For notational simplicity, from now on we will denote the argument of \(F(.)\) satisfying (3) simply by \(s_r\) (rather than \(s^*_r\)).

**Case I: \(s_t \geq s_r\)**

Whether \(s = s_t q\) will exceed \(s_t\) or not depends on the actual value of \(q\). If \(q < s_t/s_t\) then \(s > s_t q\) and accordingly to the optimal policy for the production lot size, all available remanufacturable units \(s\) are remanufactured. On the other hand, if \(q > s_t/s_t\) then \(s < s_t q = s\) and \(s = s_t q = s\) and it is optimal to dispose of \(s - s_t\) units and remanufacture exactly \(s_t\). Recall that \(g(.)\) and \(G(.)\) are the density function and distribution function of the random variable \(q\). The expected profit function, given that the production lot size will be chosen optimally as described previously, is:

\[
E[TP_e(s_t)] = v \left[ \int_0^{s_t} \left( \int_0^x f(x)dx + \int_x^s f(x)dx \right) g(q) dq + \int_{s_t}^{s_t \cap b} \left( \int_0^x f(x)dx + \int_x^{s_t} f(x)dx \right) g(q) dq \right] \quad \text{(sales revenue)}
\]

\[
- b \left[ \int_0^{s_t \cap b} \left( \int_0^x f(x)dx \right) g(q) dq + \int_{s_t \cap b} g(q) dq \right] \quad \text{(shortage cost)}
\]

\[
- c_d \left[ \int_0^{s_t \cap b} \left( \int_0^x f(x)dx \right) g(q) dq \right] + \int_{s_t \cap b} \left( \int_0^x f(x)dx \right) g(q) dq \quad \text{(disposal cost of excess remanufactured)}
\]

\[
\int_0^{s_t \cap b} (s - x) f(x) dx \quad \text{g(q) dq} \]

\[
+ \int_{s_t \cap b} (s - x) f(x) dx \quad \text{g(q) dq} \]

\[
(\text{disposal cost of excess remanufactured})
\]
\[ \begin{align*}
- c_t \int_{s_t/\eta_t}^{s_t} s g(q) dq + \frac{1}{s_t} s g(q) dq 
& \quad \text{(remanufacturing cost)} \\
- c_d \int_0^{s_t} (1-q) s_t g(q) dq 
& \quad \text{(disposal cost of non-remanufacturables)} \\
- c_d \int_{s_t/\eta_t}^{1} (s - s_t) g(q) dq 
& \quad \text{(disposal cost of excess remanufacturables)} \\
- c_s s_t - c_s s_t. 
& \quad \text{(transportation and sorting costs)}
\end{align*} \]

Rearranging terms and using (3) yields:

\[ \begin{align*}
E[TP_c(s_t)] &= (v + b + c_d) \left\{ \int_0^{s_t/\eta_t} \left[ \int_0^{s_t/\eta_t} x f(x) dx \right] g(q) dq + \int_{s_t/\eta_t}^{s_t} \left[ \int_0^{s_t/\eta_t} x f(x) dx \right] g(q) dq \right. \\
& \quad + \int_{s_t/\eta_t}^{1} s_t q [F(s_t) - F(s_t, q)] g(q) dq \right\} \\
& \quad \left. + \int_{s_t/\eta_t}^{1} s_t q F(s_t) F(s_t, q) g(q) dq \right\} \\
& \quad \left. -(c_i + c_d + c_s) s_t - bE(D) \right\} .
\end{align*} \quad (4) \]

\textbf{Case II: } \text{s} = \text{s}_t, q \text{ will be lower than } s_t \text{ regardless of the actual } q \text{ value. The expected profit function, given that the production lot size will be chosen optimally (equal to s), is:}

\[ \begin{align*}
E[TP_c(s_t)] &= v \left( \int_0^{s_t} x f(x) dx + \int_0^{s_t} s f(x) dx \right) g(q) dq 
& \quad \text{(sales revenue)} \\
- b \left( \int_0^{s_t} (s - s) f(x) dx \right) g(q) dq 
& \quad \text{(shortage cost)} \\
- c_d \int_0^{s_t} (s - x) f(x) dx g(q) dq 
& \quad \text{(disposal cost of excess remanufactured)} \\
- c_i \int_0^{s_t} s g(q) dq 
& \quad \text{(remanufacturing cost)} \\
- c_d \int_0^{s_t} (l-q) s_t g(q) dq 
& \quad \text{(disposal cost of non-remanufacturables)} \\
- c_s s_t - c_s s_t. 
& \quad \text{(transportation and sorting costs)}
\end{align*} \]

which, using (3), becomes:

\[ \begin{align*}
E[TP_c(s_t)] &= (v + b + c_d) \left\{ \int_0^{s_t/\eta_t} \left[ \int_0^{s_t/\eta_t} x f(x) dx \right] g(q) dq + \int_{s_t/\eta_t}^{s_t} \left[ \int_0^{s_t/\eta_t} x f(x) dx \right] g(q) dq \right. \\
& \quad + \int_{s_t/\eta_t}^{1} s_t q [F(s_t) - F(s_t, q)] g(q) dq \right\} \\
& \quad \left. + \int_{s_t/\eta_t}^{1} s_t q F(s_t) F(s_t, q) g(q) dq \right\} \\
& \quad \left. -(c_i + c_d + c_s) s_t - bE(D) \right\} .
\end{align*} \quad (5) \]

Since the second-order derivative of \(E[TP_c(s_t)]\) is negative for every \(s_t > 0\) it follows that \(E[TP_c(s_t)]\) is concave for every \(s_t \geq 0\). Thus, there will be a unique value of \(s_t\) which maximizes the expected profit. This unique optimal solution will be the value of \(s_t\) for which the first-order derivative vanishes. If there does not exist such point, then \(E[TP_c(s_t)]\) will be a decreasing function of \(s_t\) and the optimal value will be \(s_t^* = 0\). Specifically:
a) If \( E(q) \leq \frac{c_w + c_d + c_i}{v + b + c_d - c_r} \), then the optimal value of \( s_i \) is \( s_i^* = 0 \).

b) If \( E(q) > \frac{c_w + c_d + c_i}{v + b + c_d - c_r} \) and \( \int_0^1 q[F(s_i) - F(s_i, q)]g(q) dq \leq \frac{c_w + c_d + c_i}{v + b + c_d} \), then the optimal value of \( s_i \) is \( s_i^* \) such that:

\[
\int_0^1 q[F(s_i) - F(s_i, q)]g(q) dq = \frac{c_w + c_d + c_i}{v + b + c_d}.
\]

c) If \( E(q) > \frac{c_w + c_d + c_i}{v + b + c_d - c_r} \) and \( \int_0^1 q[F(s_i) - F(s_i, q)]g(q) dq > \frac{c_w + c_d + c_i}{v + b + c_d} \), then the optimal value of \( s_i \) is \( s_i^* \) such that:

\[
\int_0^{s_i^*} q[F(s_i) - F(s_i, q)]g(q) dq = \frac{c_w + c_d + c_i}{v + b + c_d}.
\]

4. Local sorting

When sorting takes place at CS the information regarding the quality of the returns becomes available before the transportation of the products from CS to R. The expected number of units inspected until \( s_i \) remanufacturable ones are identified is

\[
\int_0^1 \frac{1}{q} g(q) dq = s_i E(1/q).
\]

There is now a single decision variable \( s_i \equiv s_i \), and as a result the mathematical model is much simpler. The expected profit function is:

\[
E[TP_d(s_i)] = \int_0^{s_i} x f(x) dx + \int_{s_i}^{\infty} s_i f(x) dx - b \int_0^{s_i} (x - s_i) f(x) dx - c_d \int_0^{s_i} (s_i - x) f(x) dx - c_i s_i - c_i c_s - c_{id} s_i E(1/q).
\]

By rearranging terms and after some algebraic manipulation we obtain:

\[
E[TP_d(s_i)] = (v + b + c_d) \int_0^{s_i} (x - s_i) f(x) dx + [v + b - c_{id} E(1/q) - c_i - c_s] s_i - b E(D).
\]

By taking the second-order derivative it is easy to show that \( E[TP_d(s_i)] \) is concave. The unique optimal solution can be found using the first-order conditions. Specifically:

a) If \( v + b \leq c_{id} E(1/q) + c_i + c_s \), then the optimal value of \( s_i \) is \( s_i^* = 0 \).

b) If \( v + b > c_{id} E(1/q) + c_i + c_s \), then the optimal value of \( s_i \) is \( s_i^* \) such that:

\[
F(s_i^*) = \frac{v + b - [c_{id} E(1/q) + c_i + c_s]}{v + b + c_d}.
\]
5. Numerical illustration

For the case of centralized sorting, the model has been optimized for 16 different sets of parameters. The random variable \( q \) is assumed to follow the Beta distribution with density function:

\[
g(q) = \frac{(a + b - 1)!}{(a - 1)!(b - 1)!} q^{a-1} (1 - q)^{b-1}, \quad 0 \leq q \leq 1
\]

and parameters \((a=6, b=2)\) or \((a=2, b=6)\). These values of \( a \) and \( b \) lead to mean value of percent remanufacturable equal to 0.75 and 0.25 respectively and to constant standard deviation (equal to 0.14). The demand is modeled by the normal distribution with mean \( \mu=100 \) and standard deviation \( \sigma=30 \). The unit cost is defined equal to the disposal cost \( c_d=1 \). The costs of transportation \( c_t \), central sorting \( c_{sc} \) and remanufacturing \( c_r \) as well as the sales price \( v \) are examined at 2 levels each, but with \( c_r=10c_{sc} \). In all cases \( b \) is considered negligible. The leftmost columns of Table 2 contain the 8 combinations of the cost parameters. The remainder of Table 2 contains the optimization results for the cases with \( E(q)=0.75 \) and 0.25.

### Table 2: Parameter values and numerical results for the case of central sorting

<table>
<thead>
<tr>
<th>Case</th>
<th>Parameter Values</th>
<th>Optimization Results</th>
<th>E[TP(c)]</th>
<th>E[TP(c)]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( c_t )</td>
<td>( c_{sc} )</td>
<td>( c_r )</td>
<td>( v )</td>
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<tr>
<td>1</td>
<td>2</td>
<td>1</td>
<td>10</td>
<td>50</td>
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<tr>
<td>8</td>
<td>10</td>
<td>4</td>
<td>40</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 2 shows that \( E[TP(c(s)) \) increases in \( E(q) \), as expected. On the other hand, \( s^* \) may either increase or decrease when \( E(q) \) falls from 0.75 to 0.25, but the expected number of remanufacturables available at \( R, s^*_rE(q) \), increases in \( E(q) \). The optimal \( s^*_r \) can be either larger or smaller than \( s_r \). In all cases, though, \( s^*_rE(q) \) is smaller than \( s_r \), apart from the case with the most favorable parameter values, i.e., case 5 with \( E(q)=0.75 \).

In order to facilitate a comparison between the centralized and the decentralized system, we define \( r \) as the breakeven value of the ratio \( c_{sd}/c_{sc} \), for which the two systems have the same optimal expected profit, all else being equal. The values of \( r \) for the 16 sets are shown in Table 3. Note that for the cases with \( s^*_r = 0 \) and \( E[TP_d(s^*_r)]=0 \) there is a region of \( r \) values of the form \([r_{min}, +\infty)\), for which \( E[TP_d(s^*_r)]=E[TP_d(s^*_r)]=0 \); In these cases the \( r_{min} \) values are given in parenthesis. In addition, Table 3 contains the optimal production-procurement lot sizes \( s^*_r \) for the decentralized system.

### Table 3: Breakeven ratios \( r=c_{sd}/c_{sc} \) and optimal lot sizes \( s^*_r \) for the decentralized system

<table>
<thead>
<tr>
<th>Case</th>
<th>E(q)=0.75 ( s^*_r )</th>
<th>E(q)=0.25 ( s^*_r )</th>
<th>E(q)=0.75</th>
<th>E(q)=0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( r=c_{sd}/c_{sc} )</td>
<td>( s^*_r )</td>
<td>( s^*_r )</td>
<td>( s^*_r )</td>
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<tr>
<td>1</td>
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<td>114</td>
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<td>4.37</td>
<td>98</td>
<td></td>
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</tr>
<tr>
<td>3</td>
<td>1.15</td>
<td>44</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>(&lt;0)</td>
<td>0</td>
<td></td>
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</tr>
<tr>
<td>5</td>
<td>2.89</td>
<td>129</td>
<td>109</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>5.29</td>
<td>117</td>
<td>85</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1.48</td>
<td>99</td>
<td>79</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>1.95</td>
<td>91</td>
<td>26</td>
<td></td>
</tr>
</tbody>
</table>
Excluding from consideration the 4 cases with \( s'_i = 0 \), Table 3 leads to the following observations. The values of \( r \) range from 1.15 to 6.92. Large \( r \) values imply that local sorting is economically preferable to central sorting even if the sorting cost at the collection site is much higher than the respective cost when sorting is done centrally, at the remanufacturing facility. It is worth noting that the largest \( r \) value corresponds to the case with the larger transportation cost, lower yield and the most favorable combination of the remaining parameters, while the lowest \( r \) value corresponds to the case with smaller transportation cost, higher yield and the less favorable parameter combination among the cases with positive expected profit. The impact of transportation cost on \( r \) is obvious. When remanufacturing is more profitable the production quantities increase and consequently the impact of transportation cost becomes more evident. Moreover, since with local sorting the system avoids the transportation of non-remanufacturable units, the benefits of local sorting increase when yield is low. When the expected yield is high \( r \) varies between 1.15 and 5.29 with average value 2.80. For the lower expected yield \( r \) ranges from 1.21 to 6.92 averaging 3.25.

Finally, a side-by-side comparison of Tables 2 and 3 shows that the procurement - production quantity \((s'_r)\) of the decentralized system never exceeds either the procurement \((s'_r)\) or the production quantity \((s_r)\) of the respective centralized system. This relationship may be explained by the fact that in the case of central sorting a) the production lot size is determined without taking into account the transportation and sorting costs, and b) the procurement lot size must be large enough to compensate for the non-remanufacturable units.

6. Conclusion and extensions
In the present paper we have studied the issue of location of the sorting operations for returned products in a simple reverse supply chain. A preliminary numerical investigation showed that sorting at the collection site becomes more favorable compared to sorting at the remanufacturing facility as yield decreases, transportation cost increases and as higher revenues and lower costs make remanufacturing more profitable.

There are some interesting extensions of the present single-period model that are worth studying, such as the generalization to the case of multiple collection sites and the case of imperfect sorting. Other extensions include the consideration of a larger number of possible quality states of the returns and higher product complexity.

References