IMPACT OF STATISTICAL PROCESS CONTROL (SPC) ON THE PERFORMANCE OF PRODUCTION SYSTEMS- Part Two (large systems)

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Abstract: *The paper presents an approximate analytical method for evaluating the performance of long production lines in which Statistical Process Control techniques are applied. The work is an extension of the paper "Impact of Statistical Process Control (SPC) on The Performance of Production Systems-Part One" in the sense that the method proposed for the two machine production system proposed in that paper, is used as a building block to develop a new method for the analysis of longer lines.*
In the paper the relations between quality control issues and system performance are investigated for K machines production lines, with K>2 with K-1 finite capacity buffers
In order to analyze these systems a two-level decomposition approach is proposed. The original line is decomposed into K-1 sub-systems each one formed by two monitored machines and one buffer. In the paper numerical results are reported to show the accuracy of the method. Moreover, the approach is used for deriving some qualitative properties of the analyzed production systems.

Keywords: *Statistical Process Control, Performance Evaluation, Decomposition Method*

1- Introduction

In the paper the impact of Statistical Process Control (SPC) techniques on productivity performance of multi-stage production lines is investigated. Quality plays an important role when decision have to be taken about the production system configuration. Indeed, since quality control system, especially in the phase of rump-up, is normally under continuous improvement, it is very important to estimate how much the changes in the quality control system will affect the productivity performance of the system. In particular, the location of the inspection points, the number of features to be observed by one inspection station and all the other decisional variables of the quality control process have to be taken into account.
At the same time also the characteristics of the line and the system layout affect the performance of the quality control system. For example, imagine that in a *K* machine production line with *K-1* finite capacity buffers the feature processed by the first machine is monitored at the *K-th* stage; parts to be monitored have to wait into the buffers of all the *K-1* stages before they are inspected. This creates a long delay of the quality information about the state of the monitored machine and an action to repair the machine from an out of control that is shifted in time. Therefore it is not possible to design proper quality control policies without taking into account t the structure of the production system.
The proposed method have the goal of investigating the interactions between quality and productivity issues. The proposed approach uses the method proposed in [1] as a *building block* and proposes an extension of the decomposition methods [2,3,4] for evaluating the performance of the whole production line.

1-1 Literature review

Since the paper deals with both productivity and quality issues, in the analysis of the literature we consider two main areas of research.
Regarding the estimation of the system performance from a productivity point of view, approximate analytical methods based on the decomposition technique have been proposed. This technique was originally proposed by Gershwin [2] for the analysis of unreliable production lines with finite capacity buffers. Later, it has been improved for studying other machine failures structures [3], [4], other complex system architectures such as split and merge systems [5], and loop systems [6], and production control policies [7]. A review of the analytical methods based on the decomposition technique can be found in [8]. Recently, the decomposition technique has also been used by Gershwin

and Kim for estimating the performance of a quality-quantity model for long production lines [9], in which machines are subject to quality failures.
In the area of quality, the allocation of inspection effort in multistage production systems have been widely studied in literature. William and Peters [10] proposed a method for the economic design of control charts in multistage production systems. They proposed a net profit model based on the attribution of different cost coefficients at different system states. The presence of buffers in the line is not considered. A review of the inspection allocation models in multistage production lines can be found in Raz [11].
In the proposed paper, a decomposition technique is proposed to evaluate the performance of production systems in which machines are decoupled by the presence of finite capacity buffers and are monitored in their behaviour by control charts. An extension of the decomposition technique, already proposed for studying split and merge operations in production lines in [12], is needed for dealing with such complex machine behaviour. This new technique is named two level decomposition and is analyzed in chapter 3. In chapter 4 numerical results are reported to show the accuracy of the method and to derive some qualitative properties of the analyzed system and in chapter 5 conclusions and future research are treated.

2- System behavior and assumptions

The model presented in this paper is based on the description of system behavior proposed in [1]. The model however relies, at the current state of development, on some restrictive assumptions which will be detailed in the following.

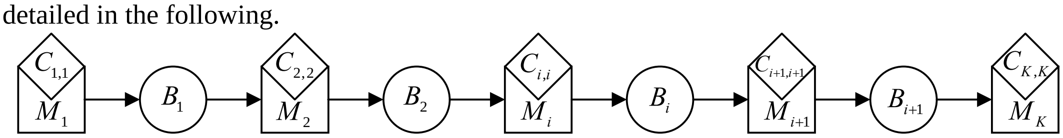

Figure 1: Representation of a K machine line with Statistical Quality Control (SPC) tools

The analyzed system is composed by K stations M_i and $(K-1)$ interoperational buffers of finite capacity N_i , $i=1,\dots,K$, storing both conforming and non-conforming parts, as represented in figure 1. Machines are unreliable and they can fail in F_i different modes. In addition to these local failures which stop the machine, machines can loose their original settings (in control state) and shift to an out of control state even if they continue to produce parts. In the paper, for each machine, a unique mode of going out of control is considered. Machines can perform machining operations, inspection operations or can integrate both the functions. Stations in which inspections are performed are composed by physical and logical components. Physical components are inspection devices, measurement equipment and actuators, while logical devices are control charts and scrap/rework policies. In this paper the possibility of scrapping and reworking parts is not considered, even if the proposed method can be extended in order to deal with this aspect. The logical components of the inspection devices represent the links between the monitored machines and the inspection stations. In the model, they are named $C_{i,q}$, where i refers to the monitored machine and q refers to the inspection station in which the logical component is located. In this paper we include only the cases in which the machines are monitored locally, i.e. inspection devices and logical components are located at the same production stage of the monitored machine ($q=i$).

3- The proposed method

The system described in the previous section is analyzed by using an extension of the decomposition technique proposed in [12], [13]. The idea of the approach is the following: the original system formed by K stations, each one monitored by control chart $C_{i,q}$ is decomposed into $K-1$ sub-systems each one formed by two-monitored machines and one buffer as described in [1]. In fact this approach differs from the classical decomposition widely used in literature. We define such a method *two-level decomposition* [12]. Indeed, the method proposed in [1] is based on the evaluation of the steady state probabilities of all the states of each of the two monitored machines. These probabilities are obtained by solving a Markov chain representing the behavior of such complex machines taking also into account the influence of their neighboring buffers. This level of decomposition is named *Machine level decomposition*. On the other hand, in this paper, the focus is on buffers. Indeed, we study failure and repair parameters of the pseudo-machines of each building block in order to traduce the flow of material through the corresponding buffer of the original line. This is a *buffer level decomposition*. By studying alternately these two levels of decomposition and passing the results obtained in one level to the other it is possible to evaluate the performance of the original complex system. In particular, the buffer level decomposition passes to the machine level decomposition the probabilities of the upstream buffer being empty (starvation probability) and the downstream buffer being full (blocking probability). From the buffer point of view, the machine level decomposition passes to each two-machine one buffer sub-system of the type analyzed in [9], the failure and repair probabilities of the upstream and downstream pseudo-machines. Two level decomposition is necessary in those cases in which machines that have complex behavior must be analyzed. For simplicity, in the rest of the paper we will consider only a unique level of decomposition, the buffer level decomposition, given the fact that the machine level decomposition is included in the approximate method proposed in [1].

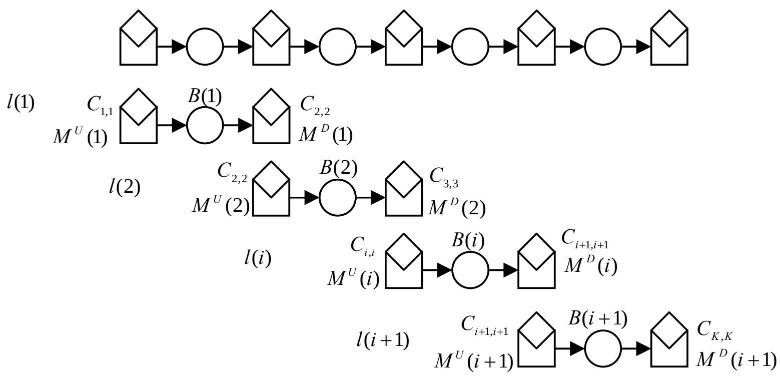

Figure 2: Decomposition of a production line with K monitored machines into building blocks

3-1 Detailed Description of the method

Following the general idea of the decomposition based on multiple failure modes [3], the pseudo-machines of each building block must be characterized so that the flow of material through the buffer of the two-machine line closely matches the flow of material through the corresponding buffer of the original line. Therefore, the upstream pseudo machine of a building block must mimic the behavior of the portion of line that is upstream the considered buffer, while the downstream pseudo-machine must mimic the behavior of the portion of line that is downstream the considered buffer. The characterization of the pseudo-machines in the building blocks consists in the attribution of remote failures modes to these machines.

Consider the upstream pseudo-machine of the generic building block $l(i)$, $M^U(i)$. The flow of material entering the buffer B_i can be interrupted for different reasons. Firstly machine M_i may fail while producing a part; this may happen both when the machine works in control and when it works out of control. Secondly machine M_i can be stopped by the control chart controlling it (either for a false alarm or for a true out of control detected). Thirdly, a machine M_j $(j=1,\dots,i-1)$, present in the portion of system upstream machine M_i may fail or may be stopped to fix quality problems. If machine M_j does not produce parts, machine M_i will continue to process parts until the upstream buffer gets empty; in this last situation machine M_i is said to be starved.

For each possible cause of interruption of flow into the buffer, a failure mode of the pseudo-machine $M^U(i)$ is defined. In particular, to capture the failures of the original line M_i, local failure modes are assigned to $M^U(i)$. Local failures can be of two types: *type f local failure modes* ($f_i=1,..,\Phi_i$), characterized by the fact that if the machine goes down, it is repaired and restored to the in control operating state independently from the state in which the machine was before the failure occurred, or *type ϕ local failure modes* ($\phi_i=\Phi_i+1,..,F_i$) where, after a failure, the machine is repaired and restored to the state it had before the failure occurred.

The probabilities of failing and repairing due to these failure modes are simply equal to those of the original machine M_i, i.e.:

$$p^U_{i,f_i}(i) = p_{i,f_i} \qquad r^U_{i,f_i}(i) = r_{i,f_i} \qquad f_i=1,..,\Phi_l \tag{1}$$

$$p^U_{i,\phi_i}(i) = p_{i,\phi_i} \qquad r^U_{i,\phi_i}(i) = r_{i,\phi_i} \qquad \phi_i = \Phi_i+1,..,F_i \tag{2}$$

To capture the situation in which the monitored machine M_i is stopped for a false alarm of out of control a failure $p_i^{false}(C_{i,q})$ is assigned to the pseudo-machine $M^U(i)$. Similarly, for capturing the situation in which the machine M_i is stopped for an observed out of control another failure mode $p_i(C_{i,q})$ is assigned to the pseudo-machine $M^U(i)$. Failure and repair probabilities of these two failure modes depend directly on the parameters of the monitoring control chart $C_{i,q}$ and can be evaluated by using the *quality link equations* introduced in [1].

Given the possibility that failures of a machine M_j positioned in the upstream portion of line propagate downstream to reach machine M_i, some additional failures, named *remote failures* [2] must be assigned to the pseudo-machine $M^U(i)$. The parameters of these failures can be obtained by means of decomposition equations. According to the method proposed in [3], remote failures of the upstream pseudo-machines can be evaluated as follows:

$$r^U_{j,f_j}(i) = r_{j,f_j} \qquad p^U_{j,f_j}(i) = \frac{Ps_{j,f_j}(i-1)}{E(i-1)} r^U_{j,f_j}(i) \qquad j=1,\ldots,i-1 \quad f_j=1,..,F_j+2 \tag{3}$$

In equation (3) all the causes of the interruption of flow due to machine M_j are included (F_j local failures of machine M_j plus one failure for false alarm of out of control plus one failure for observed and real out of control).

Since there is no reason to think that once the flow is resumed in the upstream part of line, machine $M^U(i)$ can pass to an in control state if it was in the out of control state before the remote failure occurred, remote failures are classified as *type* ϕ failures. Therefore, in the analysis of the Markov chain of the machine $M^U(i)$ (machine level decomposition) to evaluate the corresponding building block $l(i)$ [1], these failure be considered exactly as all the other type ϕ local failure modes.

The same considerations can be done for the downstream pseudo-machine of the generic building block $l(i)$, named $M^D(i)$. In this case the following equations hold:

$$r^D_{j,f_j}(i) = r_{j,f_j} \qquad p^D_{j,f_j}(i) = \frac{Pb_{j,f_j}(i+1)}{E(i+1)} r^D_{j,f_j}(i) \qquad j=i+2,\ldots,K \quad f_j=1,..,F_j+2 \tag{4}$$

In buffer $B(i)$ of the decomposed sub-line $l(i)$ both conforming and non-conforming parts are stored. Therefore the effective average throughput of the sub-system $l(i)$ is affected by the fraction of conforming parts produced at the upstream production stages. . To evaluate the flow of conforming parts entering the buffer B_i another decomposition equation is needed. This decomposition equation is necessary to calculate the yield [1] (the fraction of conforming parts) produced by the pseudo-machine $M^U(i)$.

$$Y^U(i) = \prod_{j=1}^{i} Y_j \qquad Y^D(i) = \prod_{j=i+1}^{K} Y_j \qquad Y(i) = Y^U(i) \cdot Y^D(i) \tag{5}$$

3-2 The algorithm

The proposed decomposition equations have been solved by using an algorithm inspired to the DDX algorithm [14]. This algorithm consists in the following steps:

- Phase 1: Initialization: set all the failure and repair probabilities of the local failure modes of the pseudo-machines in the building blocks to the failure and repair probabilities of the corresponding machine (M_i) in the original line. Set all the repair probabilities of the remote failures of the pseudo-machines equal to the repair probabilities of the corresponding machines (M_j) in the original line and set the failure probabilities of the remote failures to a given value $t=0.05$. Solve all the building blocks $l(i)$, for $i=1,\ldots,(K-1)$.
- Phase 2: Upstream pseudo-machines analysis: For i=$2,\ldots,K-1$ perform the following steps:
  - Step 1: Calculate the failure probabilities of the remote failures of the pseudo-machine $M^U(i)$ by using equation (3);
  - Step 2: Calculate the performance of the two monitored machines sub-system $l(i)$ by using the two level decomposition approach proposed in [1] and equation (5);
- Phase 3: Downstream pseudo-machines analysis: For $i=(K-2),\ldots,1$ perform the following steps:
  - Step 1: calculate the failure probabilities of the remote failures of the pseudo-machine $M^D(i)$ by using equation (4);
  - Step 2: calculate the performance of the two monitored machines sub-system l(i) by using the two level decomposition approach proposed in [1] and equation (5);

Applying alternately phase 2 and phase 3 of the algorithm the performance of the line can be obtained. The algorithm stops when the following conditions are satisfied (conservation of total flow and conservation of effective flow):

$$E(1) = E(2) = ... = E(K-1) \quad and \quad Y(1) = Y(2) = ... = Y(K-1) \tag{6}$$

In this situation, the average buffer levels are determined by using:

$$n_i = n(i) \qquad for\ i=1,...,K\text{-}1 \tag{7}$$

The average throughput of the system is:

$$E^{tot} = E(1) = ... = E(K-1) \tag{8}$$

And the effective throughput of the system is:

$$E^{eff} = E^{tot} \cdot Y^{tot} = E^{tot} \cdot Y(1) = ... = E^{tot} \cdot Y(K-1) \tag{9}$$

4- Numerical Results

4-1 Accuracy testing

The proposed method has been implemented and results provided have been compared with simulation. The simulation model has been built adopting the same assumption of the approximate analytical model. For each case 10 simulation runs of 5.000.000 time units have been performed. Consequently the 95% confidence interval on the average throughput estimation has a maximum width of 0.000978. In the tables cases of 3 machine lines (table 1,2), and ten machine lines (tables 3,4) are reported. Both the cases of sampling (h>m) and 100% inspections (h=0) are considered in the examples.

C	N	M_i	$p_i^{quality}$	$r_i^{quality}$	r_i^{false}	p_i	r_i	$\gamma_i Wi$	$\gamma_i Oi$	$h_i(C_{i,i})$	$m_i(C_{i,i})$	$\alpha_i(C_{i,i})$	$\beta_i(C_{i,i})$
1	**6**	**1**	0.12	0.6	0.6	0.017	0.102	0.01	0.8	5	1	0.04	0.1
	12	**2**	0.04	0.12	0.6	0.09	0.19	0.02	0.7	5	1	0.04	0.1
	/	**3**	0.07	0.29	0.98	0.103	0.58	0.02	0.68	5	1	0.04	0.1
2	**4**	**1**	0.06	0.23	0.7	0.016	0.42	0.02	0.12	140	20	0.027	0.08
	4	**2**	0.02	0.08	0.5	0.21	0.23	0.01	0.8	140	20	0.027	0.06
	/	**3**	0.12	0.03	0.6	0.12	0.45	0.001	0.8	140	20	0.027	0.12
3	**25**	**1**	0.001	0.12	0.2	0.006	0.42	0.001	0.08	200	20	0.027	0.003
	25	**2**	0.004	0.12	0.29	0.00429	0.0109	0.01	0.02	200	20	0.027	0.003
	/	**3**	0.003	0.02	0.33	0.021	0.092	0.001	0.042	200	20	0.027	0.003
4	**25**	**1**	0.001	0.012	0.2	0.21	0.006	0.001	0.08	200	20	0.027	0.003
	25	**2**	0.004	0.12	0.29	0.142	0.0109	0.01	0.02	200	20	0.027	0.003
	/	**3**	0.003	0.02	0.33	0.235	0.092	0.001	0.042	200	20	0.027	0.003

Table 1: data for reported three monitored machine cases

CASES		E^{tot}	E^{eff}	Yield	N_1	N_2
1	Sim.	0. 5499	0.2618	0.4761	4.265	2.366
	An.	0.5467	0.2604	0.4763	4.246	2.471
	Err %	**0.581**	**0.534**	**0.042**	**0.316**	**0.875**
2	Sim.	0.4776	0.2432	0.5092	3.406	1.229
	An.	0.4768	0.2427	0.5091	3.409	1.227
	Err %	**0.167**	**0.205**	**0.019**	**0.075**	**0.05**
3	Sim.	0.5748	0.5567	0.9686	19.302	11.706
	An.	0.5753	0.5574	0.9687	19.71	11.782
	Err %	**0.086**	**0.125**	**0.01**	**1.63**	**0.304**
4	Sim.	0.0273	0.0269	0.9866	3.443	0.344
	An.	0.0271	0.0268	0.9868	3.414	0.313
	Err %	**0.732**	**0.37**	**0.02**	**0.116**	**0.124**

Table 2: results for three monitored machine cases

C	N	M_i	$p_i^{quality}$	$r_i^{quality}$	r_i^{false}	p_i	r_i	$\gamma_i Wi$	$\gamma_i Oi$	$h_i(C_{i,i})$	$m_i(C_{i,i})$	$\alpha_i(C_{i,i})$	$\beta_i(C_{i,i})$
1	**4**	**i**	0.06	0.42	0.65	0.01	0.1	0.02	0.35	15	1	0.0027	0.018
2	**6**	**i**	0.01	0.5	0.8	0.019	0.3	0.05	0.5	0	1	0.0027	0.09

Table 3: data for ten monitored machine cases.

CASES		E^{tot}	E^{eff}	Yield	N_1	N_2	N_3	N_4	N_5	N_6	N_7	N_8	N_9
1	Sim.	0.5522	0.0849	0.1538	2.892	2.604	2.381	2.184	1.997	1.809	1.616	1.393	1.106
	An.	0.5559	0.0854	0.1538	2.9	2.618	2.317	2.179	2	1.82	1.682	1.381	1.099
	Err%	**0.67**	**0.58**	**0**	**0.2**	**0.35**	**1.6**	**0.125**	**0.075**	**0.275**	**1.65**	**0.3**	**0.175**
2	Sim.	0.8025	0.4566	0.569	4.035	3.650	3.401	3.188	2.990	2.785	2.584	2.336	1.951
	An.	0.8062	0.4588	0.5692	4.048	3.648	3.357	3.189	2.999	2.81	2.642	2.351	1.951
	Err%	**0.46**	**0.481**	**0.035**	**0.21**	**0.033**	**0.73**	**0.016**	**0.15**	**0.416**	**0.96**	**0.25**	**0**

Table 4: results for ten monitored machine cases.

As it can be noticed the proposed method is very accurate, both in terms of throughput and average buffer level.

4-2 Application example

We now simulate the application of the method for the estimation of the performance of a three machine line in which machines are monitored by using a X(bar) control chart. In the example we will consider that machines are identical and that identical control charts are used at each production stage.

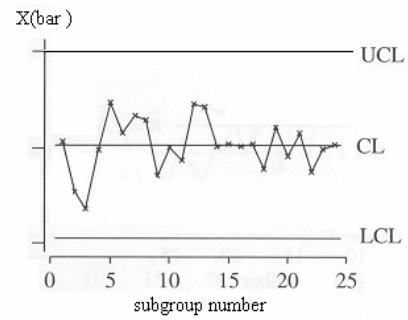

Figure 3: X(bar) control chart.

The dimension of the feature produced by machine M_i is assumed to follow a normal distribution $N(\mu_i, \sigma_i)$, equal for each monitored machine $(i=1,..,3)$. When the monitored machine is in control the mean value of the quality characteristic is $\mu_i = \mu_{0,i}$. When an out of control occurs this value shifts of the quantity δ_i expressed in standard deviation units $(\mu_i = \mu_{1,i} = \mu_{0,i} + \delta_i \ \sigma_i)$. This event occurs with probability $p_i^{quality}$. X(bar) control chart are compiled by measuring the quality characteristic X_i processed at stage i and calculating the average value $X_i(bar)$ of the quality characteristic in the measured sample of size m $(X^1_{i,} X^2_{i}, ..., X^m_{i})$. The test used to decide at each sample whether or not the machine is in out of control state has the following hypotheses:

$$H_0 : \mu_i = \mu_{0,i}$$

$$H_1 : \mu_i \neq \mu_{0,i}$$

In the control chart three lines are plotted, figure 3. One line represent the average value of the normal distribution for the quality characteristic when the monitored machine is in control, named Central Line (CL). One line is the Upper Control Limit (UCL) and one line is the lower control limit (LCL). The position of these limits in the control chart are evaluated by using the following equations:

$$UCL = \mu_{0.i} + K_i \frac{\sigma}{\sqrt{m}}$$

$$LC = \mu_{0.i}$$

$$LCL = \mu_{0.i} - K_i \frac{\sigma}{\sqrt{m}}$$

In the control chart two regions are created by these lines. The region inside the control limits is the region in which the hypothesis H_0 cannot be rejected, while the regions outside the control limits are the regions in which hypothesis H_0 can be rejected. The probability of type I and type II errors can be calculated by using the following equations:

$$\alpha_i = 2\Phi(-k_i) \tag{10}$$

$$\beta_i = p\left(Z \le \frac{UCL - \mu_{1,i}}{\sigma/\sqrt{m_i}}\right) - p\left(Z \le \frac{LCL - \mu_{1,i}}{\sigma/\sqrt{m_i}}\right) = \Phi(k_i - \delta_i\sqrt{m}) - \Phi(-k_i - \delta_i\sqrt{m}) \tag{11}$$

Where Φ is the cumulative normal distribution function. Table 5 reports data of the example case (machines M_i and control charts $C_{i,i}$ have the same parameters at each production stage). In the example $k_i(C_{i,q})=3$ for each machine, therefore control charts are standard Shewart control charts.

Case	N	$p_i^{quality}$	$r_i^{quality}$	r_i^{false}	p_i	r_i	$h_i(C_{i,i})$	$m_i(C_{i,i})$	$k_i(C_{i,i})$	$\delta_i(C_{i,i})$	$\alpha_i(C_{i,i})$	$\beta_i(C_{i,i})$
1	**4**	0.03	0.12	0.6	0.01	0.1	20	5	3	0.5	0.0027	0.97
2	**4**	0.03	0.12	0.6	0.01	0.1	20	5	3	1	0.0027	0.777
3	**4**	0.03	0.12	0.6	0.01	0.1	20	5	3	1.5	0.0027	0.3616
4	**4**	0.03	0.12	0.6	0.01	0.1	20	5	3	2	0.0027	0.07

Table 5: data of the example case.

The evaluation of the system performance is made by following this procedure:

- Evaluate the type I and the type II error probabilities of each control chart by using (10) and (11);
- Evaluate probabilities of detecting an out of control p_i $(C_{i,q})$ and probability of having a false alarm detected $p_i^{false}(C_{i,q})$ by using *quality link equations* proposed in [1];
- Evaluate the performance of the system by following the method proposed in this paper and the solution of building blocks proposed in [1].

The results of this procedure are shown in table 6.

Case		E^{tot}	E^{eff}	Yield	N_1	N_2
1	Sim	0.7749	0.4388	0.5663	2.353	1.632
	An	0.7757	0.439	0.5658	2.377	1.626
	Err	**0.103**	**0.045**	**0.088**	**0.6**	**0.15**
2	Sim	0.721	0.4256	0.5903	2.37	1.637
	An	0.7215	0.4252	0.5893	2.401	1.616
	Err	**0.069**	**0.093**	**0.169**	**0.775**	**0.525**
3	Sim	0.6591	0.4109	0.6234	2.38	1.617
	An	0.6596	0.4105	0.6223	2.421	1.601
	Err	**0.075**	**0.097**	**0.176**	**1.025**	**0.4**
4	Sim	0.6357	0.406	0.6387	2.381	1.605
	An	0.6355	0.4051	0.6375	2.427	1.594
	Err	**0.031**	**0.221**	**0.187**	**1.15**	**0.275**

Table 6: results of the example case.

4-3 Qualitative properties of the system

Given the accuracy of the proposed method, we use it to analyze some properties of the system. For the three monitored machine line with parameters reported in table 7 (equal for each machine and each control chart in the line) we investigate how the performance measures varies while varying one by one the input parameters of the first machine in the line.

$p_i^{quality}$	$r_i^{quality}$	r_i^{false}	p_i	r_i	$\gamma_i Wi$	$\gamma_i Oi$	$h_i(C_{i,i})$	$m_i(C_{i,i})$	$\alpha_i(C_{i,i})$	$\beta_i(C_{i,i})$
0.001	0.12	0.5	0.001	0.01	0.01	0.03	20	3	0.01	0.01

Table 7: data for the example case.

Figure 4 reports the impact of the number of part between samples $h_i(C_{i,i})$ on the average buffer levels. By increasing $h_i(C_{i,i})$ the out of control state of the machine is identified with longer response time of the control chart. This leads to an increasing total throughput and an increasing average number of parts in the buffers, following a concave function. The effect of the increasing of the probability of having a shift to an out of control state is a decreasing of both the total throughput of the system (since machine need to be repaired more frequently from the out of control) and of the system yield (since the machine passes more in the out of control state). Also, the average level of buffers decreases, since machine inserting material in the buffer 1 is more frequently failed, figure 5 and 6.

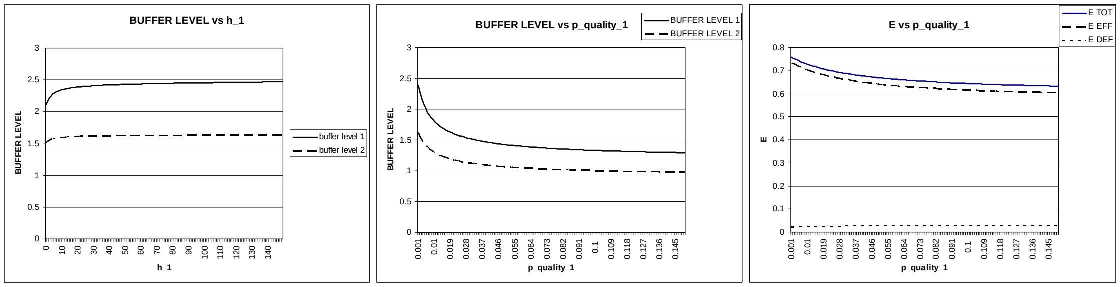

Figure 4,5,6: effect of $h_i(C_{i,i})$ and $p_i^{quality}$ on the system performance measures.

5- Conclusions

An approximate analytical method for evaluating the performance of a production system in which machines are monitored by using Statistical Process Control theory has been developed. In particular, the method considers the presence in the line of inspection points which collect data to compile control charts. Numerical results show that the method is accurate both in evaluating the system throughput and the yield. The following step will be to model the scrapping or reworking of non-conforming parts. Also, the possibility of realizing inspections downstream the monitored machine will be included. This approach will show the effect of buffers on the delay of the quality information and on the performance of the quality control system in terms of responsiveness. As a future development, models for the optimal design of control charts, optimal allocation of inspection stations and of buffers capacity will be developed. Finally an optimization procedure can be proposed to take into account all the three aspects together in order to find an optimal solution that minimizes a cost function.

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