

INTEGRATED OPTIMIZATION OF SAFETY STOCK AND TRANSPORTATION CAPACITY

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Abstract:

We consider a segment of a supply chain comprising an inventory system and a transportation system that cooperate in the fulfillment of stochastic customer orders. The inventory system is operated under a discrete time (s, q) policy with backorders. In a system of this kind accumulated backorders introduce lumpiness of transportation demand into the system. We study the connection between the safety stock and the number of available transportation resources, for example the vehicle fleet size.

Keywords:

Inventory; vehicle fleet size.

Introduction

We consider an inventory node in a supply chain facing stochastic demand for a single product. An (s, q) (reorder point, reorder quantity) inventory policy with review at the end of each period is applied. Customers arrive periodically on a discrete time axis. Period demands are stationary and i. i. d. and may follow any probability distribution. Each period demand is composed of a stochastic number of customer orders. Unfilled customer orders are backordered. In addition to the inventory process we consider the transportation process as a second time-consuming phase in the distribution of the product to its customers. Under these conditions, the waiting time that a customer order observes is composed of two parts with different reasons: *waiting for inventory service* in case of a stockout and *waiting for transportation service* in case of finite transportation capacity. The performance of the studied segment of a supply chain is measured in terms of the customer order waiting time, which is the sum of these two times.

After a customer order has been processed in the inventory system, it is passed to the material handling and transportation system, which has a limited number of specialized vehicles (such as tankers or refrigerator trucks) operated by the company. Each vehicle k is able to deliver a random number V_k of orders per period. The situation described has been observed in a large chemical company.

We consider a typical inventory order cycle. The delivery process usually runs as follows. Starting at the point in time, when a replenishment order has arrived, first all accumulated inventory backorders are processed by the inventory system and passed to the transportation system for delivery. For each inventory backorder a transportation order is released, which joins the queue in front of the transportation system. As the transportation capacity is limited, at the end of any period there may be some transportation orders postponed till the next period. These orders are called *transportation backorders*.

After all transportation backorders have been delivered, the normal delivery process runs with a stochastic number of transportation orders that depend on the arrival of customer demands in the inventory system. The inventory cycle ends with a number of periods, when the on hand inventory has fallen to zero and no further transportation orders arrive at the transportation system.

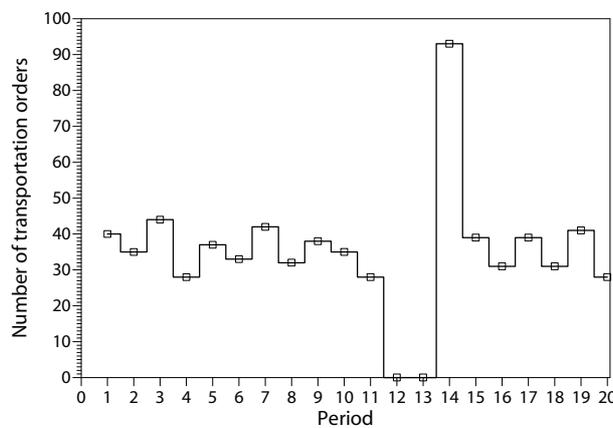


Figure 1: Order arrival in the transportation department

Caused by the availability of stock in the inventory system, the demand for transportation capacity forms a sequence of periods with very high demand, followed by a number of periods with normal demand, and concluded by a number of periods with no demand. After a replenishment order has arrived from the outside supplier, first all inventory backorders are delivered generating a peak in the transportation demand. The arrival process of orders in the transportation department is depicted in Figure 1.

Figure 2 illustrates the simultaneous development of inventory backorders and transportation backorders over time. The graph at the top shows the inventory backorders observed at the end of an order cycle. The graph at the bottom depicts the transportation backorders observed at the end of each period (day).

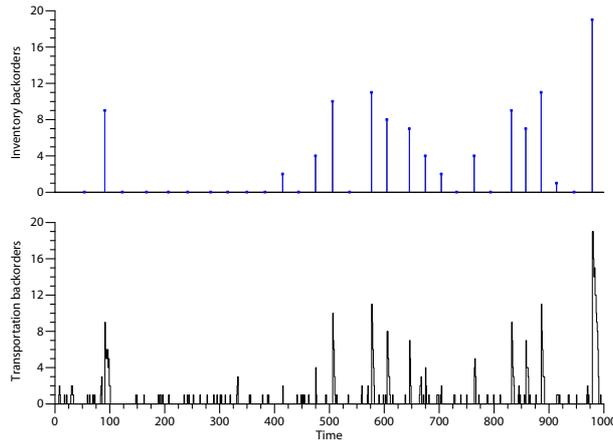


Figure 2: Simultaneous development of inventory and transportation backorders

Note that a low service level in the inventory system is associated with a large amount of backorders waiting at the end of an order cycle. This increases the variability (lumpiness) of demand for transportation capacity. If the latter is limited, then significant delays in the transportation process may arise.

Assumptions and symbols

Our analysis is based on the following assumptions:

1. Time is divided into discrete units, e. g. days or weeks, with periodic arrivals of demands and periodic inventory reviews.
2. Period demand quantities are independent and identically distributed random variables. Each period demand quantity is composed of a stochastic number of orders.
3. In the inventory system, a (s, q) policy is in place.
4. Customer orders are processed in the inventory system and forwarded to the transportation system, which delivers them to the customers. Each transportation order that is not processed on the day of its arrival in the transportation system becomes a transportation backorder and observes a *transportation waiting time*.
5. Unmet demands in the inventory system are backordered and observe an *inventory waiting time*.
6. The inventory replenishment lead time ℓ is deterministic.
7. Due to high setup times and/or setup costs the replenishment lot sizes q are large.
8. The transportation capacity is large enough to clear all outstanding backorders within a single inventory order cycle. This means that the maximum waiting time for transportation is the length of the order cycle.
9. The sequence of events in a given period is: arrival of a replenishment order; processing of all backorders by the inventory system; passing the backorders to the transportation system; arrival

of period demand; processing of demand by the inventory system; passing the orders to the transportation system; delivery of all orders by the transportation system in the sequence of their arrival; review of the inventory position and triggering of a replenishment order, if required.

10. Decision variables are the reorder point s and the transportation capacity v . The order size q is assumed to be given.

A_t	number of orders passed to the transportation system in period t
B_I	number of backorders of the inventory system (in terms of customer orders)
G_t	number of transportation backorders remaining at the end of period t
N	coverage (number of periods) of the reorder point s
V	transportation capacity per period
\hat{w}	average target total waiting time
\hat{w}_I	average target waiting time in the inventory system
W_I	waiting time in the inventory system
\hat{w}_T	average target waiting time for transportation
W_T	waiting time for transportation

Table 1: Symbols used

Related literature

Only a limited number of papers are available that focus on the connection between inventory and transportation decisions in the context of inventory theory. The majority of these papers study the option of using faster transportation modes in order to shorten the *replenishment lead time* and to reduce the amount of safety stock required. For example, see *Allen et al. (1985)* and *Tyworth (1992)*, who focus on the influence of the shipping time on the lead time demand. *Geunes and Zeng (2001)* consider backlogging of excess demand as an alternative to expediting in a two-stage a distribution system. Their decision variables are the safety stock and the in-house transportation capacity (expressed in product units shipped). In addition, they allow for unlimited external transportation capacity that is available at comparatively high costs. *Henig et al. (1997)* treat the joint optimization of a periodic inventory policy and a prepaid basic transportation capacity determined by a contract. In order to handle high transportation requirements an unlimited amount of transportation capacity is available at additional costs. *Yano and Gerchak (1989)* consider the simultaneous determination of safety stock and inbound transportation capacity for an assembly plant. As the "customer" is an assembly line with 100% service requirements, in case of over-utilization of the contracted transportation capacity costly emergency shipments are used. All these papers do not allow for the possibility of waiting for transportation, which is the focus of the current work.

Analysis

Denote with A_t the number of transportation orders passed from the inventory system to the transportation system in period t as a result of the processing of normal orders delivered without delay (i. e. excluding backorders) by the inventory system. Then the sequence $\{A_t, (t = 1, 2, \dots)\}$ is a stochastic process that is active during the periods when the on hand inventory is greater than zero.

As the transportation capacity per period is limited, a queue of transportation orders builds up and disappears over time. Let V denote the available transportation capacity in terms of the number of orders delivered per period (e. g. the number of vehicles, if each vehicle delivers one order per period). Note that V may be a random variable, which is the result of the operational routing decisions with respect to the locations of the customers.

Assume first that the transportation capacity is deterministic $V = v$. Then the development of the queue length is the result of the arrivals and departures of transportation orders sent from the inventory system to the transportation system. The queue length G_t of transportation backorders at the end of period t evolves over time as follows:

$$G_t = \max[G_{t-1} + A_t - v, 0] \quad t > 0 \quad (1)$$

The sequence $G_t, (t = 1, 2, \dots)$ is a discrete parameter Markov chain. A typical delivery cycle starts at time $t = 0$ immediately after the arrival of a replenishment order in the inventory system. At this point in time the inventory backorders B_I are processed and passed to the transportation system, where they become the transportation orders available for delivery in the same period. Therefore we have $P\{G_0 = i\} = P\{B_I = i\}, (i = 1, 2, \dots)$. Note that the initial conditions of the Markov chain representing the status of the transportation system depend on the delivery performance and the associated reorder point s of the inventory system. With a high safety stock level only a small number of inventory backorders will be passed to the transportation system after the inventory replenishment. If, by contrast, the safety stock level is low, then the number of inventory backorders requiring transportation to their destination is significantly greater. In fact, their number may be so high that it takes the transportation system several periods to deliver them all.

Under the assumptions stated, the probability distribution $P\{B_I = i\}$ can be approximated as follows. See also (*Tempelmeier, 1985*). Let N be the coverage (i. e. the number of periods whose demands are completely filled from on hand inventory) of the reorder point s and let ℓ be the replenishment lead time. If $N = n$, then the period demands of the periods $n + 1, n + 2, \dots, \ell$ are backordered. The cumulated number of customer orders that are backordered is the total number of orders that arrived within the last $(\ell - n)$ periods before the inventory replenishment, $B^{(\ell-n)}$. Given the probability distribution of the

number of orders per period, $P\{A = a\}$, we get

$$P\{B_I = i\} = \sum_{n=1}^{\ell} P\{B^{(\ell-n)} = i | N = n\} \cdot P\{N = n\} \quad i = 1, 2, \dots \quad (2)$$

and

$$P\{B_I = 0\} = P\{N \geq \ell\} \quad (3)$$

where $P\{B^{(\ell-n)} = i | N = n\}$ can be computed as the $(\ell - n)$ -fold convolution of the probability distribution of the number of orders per period, $P\{A = a\}$.

Now consider the process of order fulfillment in the transportation system. Let J be the maximum number of transportation backorders that will possibly occur. The conditional transition probabilities between the states of the transportation backorder queue in two consecutive periods $t - 1$ and t ($t = 1, 2, \dots$) under the assumption of a given transportation capacity $V = v$ are then

$$P\{G_t = 0 | G_{t-1} = i\} = P\{A_t \leq v - i\} \quad i = 0, 1, \dots, J \quad (4)$$

$$P\{G_t = j | G_{t-1} = i\} = P\{A_t \leq v + j - i\} \quad i = 0, 1, \dots, J; 0 < j < J \quad (5)$$

$$P\{G_t \geq J | G_{t-1} = i\} = P\{A_t \geq v + J - i\} \quad i = 0, 1, \dots, J \quad (6)$$

Now assume that the period transportation capacity V is a discrete random variable with minimum value v_{\min} and maximum value v_{\max} . Then considering all possible values of V we get

$$u_{ij} = \sum_{v=v_{\min}}^{v_{\max}} (u_{ij} | V = v) \cdot P\{V = v\} \quad i = 0, 1, \dots, J; j = 0, 1, \dots, J \quad (7)$$

where $(u_{ij} | V = v)$ is the conditional transition probability defined by (4)–(6).

In order to characterize the waiting times of the transportation orders we study the development of the transportation backorder queue during the complete order cycle. Let $U^{(t)}$ be the t -step transition matrix. Then the probabilities for the number of transportation orders at the end of period t waiting for transportation in the next period are

$$P\{G_t = j\} = \sum_{i=0}^J P\{G_0 = i\} \cdot u_{ij}^{(t)} \quad j = 0, 1, 2, \dots, J; t = 1, 2, \dots \quad (8)$$

Denote with c the length of a typical inventory cycle. The average waiting time for transportation is then

$$E\{W_T\} = \frac{1}{c} \cdot \sum_{t=1}^c \frac{E\{G_t\}}{E\{A\}} \quad (9)$$

The waiting time for transportation depends primarily on the accumulated number B_I of inventory backorders at the beginning of a new cycle, immediately after the arrival of a replenishment order at the inventory system, and on the transportation capacity v that is available to reduce this backlog over time. Therefore we have

$$W_T = f[v, B_I(s)] \quad (10)$$

Note that $B_I(s) = G_0$ is a random variable that is a function of the inventory policy. In the assumed case of an (s, q) policy with fixed order quantity q , the sole influence factor is the reorder point s .

For a given number of inventory backorders B_I , there exists a stepwise inverse relationship between the transportation capacity and the average waiting time of a customer order for transportation, as shown in Figure 3.

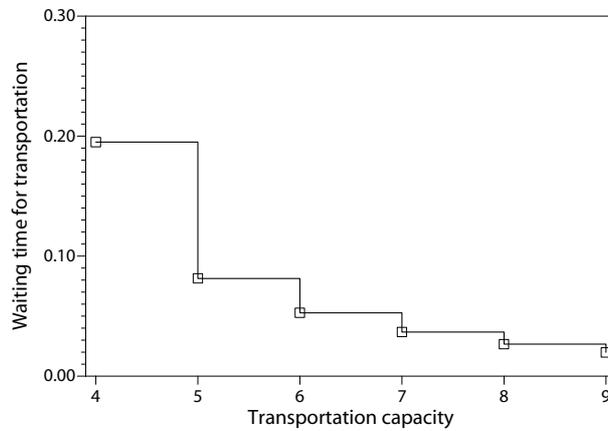


Figure 3: Waiting time for transportation versus transportation capacity

Numerical illustration

Consider an (s, q) -inventory system where the period demand follows a gamma distribution of order $n = 1.335816$ and with scale parameter $\lambda = 0.008782$, such that the expectation is $\frac{n}{\lambda} = 152.11$ and the standard deviation is $\sqrt{\frac{n}{\lambda^2}} = 131.61$. Each period demand is considered as a single order (i. e. $A = 1$). Given an order quantity $q = 5000$, a deterministic replenishment lead time $\ell = 15$ and a target average inventory-related waiting time $\hat{w}_I = 0.13$, the reorder point is $s = 2254$. The computed probabilities of the number of inventory backorders and the values observed with an ARENA simulation model are put together in the Table 2.

j	computed	simulated
0	0.4231	0.4117
1	0.1184	0.1174
2	0.1173	0.1171
3	0.1060	0.1069
4	0.0867	0.0893
5	0.0636	0.0677
6	0.0416	0.0430
7	0.0239	0.0264
8	0.0119	0.0134
9	0.0051	0.0044
10	0.0018	0.0021
11	0.0005	0.0005
12	0.0001	0.0001

Table 2: Probability distribution of inventory backorders at the arrival of a replenishment order

If we assume that the vehicle capacity is random with $P\{V = 1\} = 0.5$ and $P\{V = 2\} = 0.5$ (Case 1) and with $P\{V = 1\} = 0.7$, $P\{V = 2\} = 0.2$ and $P\{V = 3\} = 0.1$ (Case 2), we get the probabilities of transportation backorders shown in Table 3.

j	Case 1		Case 2	
	computed	simulated	computed	simulated
0	0.8981	0.8988	0.8568	0.8565
1	0.0324	0.0319	0.0434	0.0428
2	0.0250	0.0247	0.0343	0.0340
3	0.0180	0.0180	0.0254	0.0261
4	0.0120	0.0121	0.0174	0.0178
5	0.0073	0.0072	0.0110	0.0114
6	0.0040	0.0039	0.0062	0.0062
7	0.0020	0.0020	0.0032	0.0032
8	0.0008	0.0009	0.0014	0.0014
9	0.0003	0.0003	0.0005	0.0005
10	0.0001	0.0001	0.0002	0.0002

Table 3: Distribution of transportation backorders at the end of a period

Optimization

Assume that the complete transportation capacity is decided upon in advance and remains unchangeable. In this case, a customer order may suffer from two different types of delays, actually a delay caused by a stockout situation in the inventory system and a delay caused by insufficient transportation capacity. The customer usually does not care about the the type of delay. He is only interested in the overall delay that he uses as the performance measure of the supply chain.

Given a target overall average waiting time of a customer order, \hat{w} , we now seek the cost-optimum combination of the decision variables, namely the target average inventory waiting time, \hat{w}_I , and the target average transportation waiting time, \hat{w}_T , and the associated reorder point s and transportation capacity V . As we assume that the replenishment order size q is given, the waiting time observed by a customer order in the inventory system is a function of the reorder point s . To put it the other way round, for any given target waiting time \hat{w}_I the reorder point s is a function $s(\hat{w}_I)$. Let c_h denote the inventory holding cost. Then the costs related to the reorder point are $C_I = c_h \cdot s(\hat{w}_I)$.

The cost of the transportation system are a function of the transportation capacity V . Assume that the transportation capacity is a function of the number of vehicles k , $V(k)$. In the case that a vehicle can deliver one order per period, we have $V(k) = k$. If the fixed cost per period of a vehicle are c_k , the cost of the transportation system per period are $C_T = c_k \cdot k$. Due to the indivisibility of vehicles, the waiting time for transportation is a step-wise non-increasing function $\hat{w}_T(k)$ (see Figure 3). Therefore

the following optimization problem can be formulated:

$$\text{Minimize } C(\hat{w}_I, k) = c_h \cdot s(\hat{w}_I) + c_k \cdot k \quad (11)$$

s. t.

$$\hat{w}_I + \hat{w}_T(k) = \hat{w} \quad (12)$$

$$k \text{ integer} \quad (13)$$

This is a two-dimensional nonlinear optimization problem. Substituting $\hat{w}_I = \hat{w} - \hat{w}_T(k)$ we get

$$\text{Minimize } C(\hat{w}_I, k) = c_h \cdot s[\hat{w} - \hat{w}_T(k)] + c_k \cdot k \quad (14)$$

s. t.

$$0 \leq \hat{w}_T(k) \leq \hat{w} \quad (15)$$

$$v \text{ integer} \quad (16)$$

This is a nonlinear one-dimensional optimization problem in the variables k with an embedded optimization problem of finding the minimum reorder point s for any given waiting time $[\hat{w} - \hat{w}_T(k)]$ in the inventory system. The optimum solution of the above problem can be found by enumeration of all numbers of vehicles k that are feasible.

Conclusion

Supply chain management in the narrow sense requires the consideration of all time-consuming subprocesses in the process of delivery of good to the customers. The overall perspective enables the planner, to shift performance from one subprocess to the other, based on the consideration where a specific degree of performance can be achieved at the lowest costs.

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