MODELING AND PROPERTIES OF GENERALIZED KANBAN CONTROLLED ASSEMBLY SYSTEMS[1]

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Abstract:
In this paper, we are interested in production systems, realizing an assembly operation between components, and controlled by a Generalized Kanban policy. We define precisely how the Generalized Kanban mechanism can be applied in the case of assembly systems. Two different control mechanisms, already introduced for Kanban systems, can be used for controlling the flow of parts. In one case, kanbans attached to components are released simultaneously when the assembly occurs, whereas in the other case, they are released independently. We then model these Generalized Kanban controlled assembly systems as queueing networks with synchronization mechanisms and propose some properties for these models.

Keywords:
Pull systems, Generalized Kanban Policy, Assembly, Simultaneous release, Independent release, Queueing networks.

I. INTRODUCTION

We are interested in pull controlled production systems, with a make to stock production. Generally, in order to implement efficiently a pull policy, intermediate storage points are chosen, in addition to the finished parts buffer. Then the systems are decomposed into stages, each stage consisting of a production subsystem producing semi-finished parts. There are many ways to implement a pull mechanism, among which Kanban and Base-Stock control systems are the best known and the simplest to understand and implement. More general pull control policies have been proposed and a comparison of some of them can be found in [LIB 2000] and in [BOL 2004]. Among these, we consider here the special case of the Generalized Kanban control policy, introduced by Buzacott [BUZ 1989], which includes Base-Stock and Kanban control policies as particular cases. The advantage of this policy has been already shown in previous works, for systems having stages in series, [FRE 1995], [DUR 1997], [DUR 2000]. In this paper, we are interested in manufacturing systems realizing assembly. Note that although assembly systems are of particular interest in manufacturing, only a few works are devoted to the analysis of pull controlled assembly systems [SBI 2000].

The purpose of this paper is to study how the Generalized Kanban policy can be applied for assembly systems in order to coordinate production. In comparison with the case of stages in series, there is an additional difficulty consisting in the necessity to synchronize several stages together. Thus we have to define precisely how the Generalized Kanban mechanisms can be applied in the case of assembly systems. Two different assembly strategies, already introduced for Kanban systems [DI 1996], [MAT 2005] and for Extended Kanban systems [CHA 2000] can be used to control the flow of parts: the assembly with simultaneous release and the assembly with independent release. In this work, we propose one model for each of these two assembly strategies; these models can be used for systems performance evaluation. Then, for each of these two models, we establish several properties,

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concerning more particularly the bounds relative to the number of customers in the different queues of the system. Then we compare the time evolutions for both models. Therefore, for each of the two models, we are interested in the system behavior when we modify its parameters values.

II. MODELING OF GENERALIZED KANBAN CONTROLLED ASSEMBLY SYSTEMS

Let us present the principle of Generalized Kanban controlled assembly systems through a simple example inspired from [DI 1996]. We consider a production system with six machines M_1, M_2, M_3, M_4, M_5 and M_6 (see figure 1). This production system is decomposed into three stages. Each stage consists of a manufacturing process and an output buffer O_i (i = 1, 2, 3). The finished parts of the production system are in the output buffer O_3. A part can either be waiting for or receiving service at one of the different machines. This system is supplied with two types of raw parts. The first type of raw parts is successively processed on M_1, M_2 and M_3 whereas the second type of raw parts is successively processed on M_4 and M_5. A finished part is obtained by assembling two semi-finished parts (or components) that are provided by machines M_3 and M_5 respectively, and then by processing the assembled product on M_6, in order to obtain the finished product that satisfies external demands. In the following, it is assumed that raw parts are always available at the input of the system.

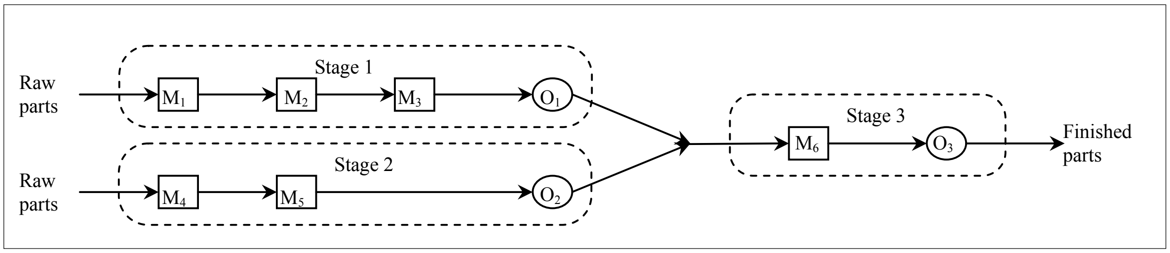

Figure-1. Example of an assembly production system.

Two different assembly strategies can be used: the assembly with simultaneous release and the assembly with independent release. For each of these two strategies, we propose a queueing network model. The queueing network model corresponding to the Generalized Kanban controlled assembly system with a simultaneous release (GKS) is represented in figure 2. The queueing network model corresponding to the Generalized Kanban controlled assembly system with an independent release (GKI) is represented in figure 3. For these two models, MP_1 models the manufacturing process of stage 1; it is constituted by machines M_1, M_2 and M_3. In the same way, MP_2 models the production mechanism of stage 2; it is constituted by two serial queues corresponding to machines M_4 and M_5. Finally, MP_3 models the production mechanism of stage 3; it consists in one queue corresponding to machine M_6. MP_1, MP_2 and MP_3 are represented by ovals on figures 2 and 3. Queues P_i (i = 1, 2, 3) model the output buffers O_i and contain consequently the stage-i finished parts. Queues B_i (i = 1, 2, 3) contain the stage-i available kanbans. At the initial state, each queue P_i contains S_i stage-i finished parts and the K_i stage-i kanbans are in queue B_i. Queue A_4 receives the transfer authorizations of stage-3 finished parts towards the consumer. Queue D_4 contains the production demands for stage 3, sent out by the consumer. Except for queues P_i and B_i (i = 1, 2, 3), all the other queues are empty at the initial state.

II.1. Queueing model for assembly systems with simultaneous release (GKS)

In the case of an assembly operation with simultaneous release (see figure 2), production demands for stages 1 and 2, sent out by stage 3, are backordered in queue D_3. Queue A_3 receives transfer authorizations of one stage-1 finished parts and one stage-2 finished part, towards stage 3.

II.2. Queueing model for assembly systems with independent release (GKI)

In the case of an assembly operation with independent release (see figure 3), production demands for stages 1 and 2, sent out by stage 3, are backordered in queues D_{31} and D_{32} respectively. Queue A_3 receives transfer authorizations of one stage-1 finished parts and one stage-2 finished part, towards stage 3.

II.3. Behaviour of GKS and GKI systems

For both types of assembly systems, when a part has finished its processing in MP_i ($i = 1, 2, 3$), the kanban is detached from the part and is put in queue B_i , whereas the obtained finished part is put in queue P_i. When an external demand arrives in the system, it causes simultaneously the supply of queues A_4 and D_4. This authorizes the consumption of one stage-3 finished part by the consumer on the one hand (if queue P_3 is non empty, otherwise this authorization is backordered in queue A_4 until P_3 is supplied), and constitutes a production demand for stage 3 on the other hand; if there is at least one available stage-3 kanban in queue B_3, (if queue B_3 is empty, this demand is backordered in queue D_4 until the arrival of a new stage-3 available kanban in queue B_3). The new production demand is transmitted upstream as a kanban coupled with a demand. Then, according to the adopted assembly strategy, the behavior of the system is different.

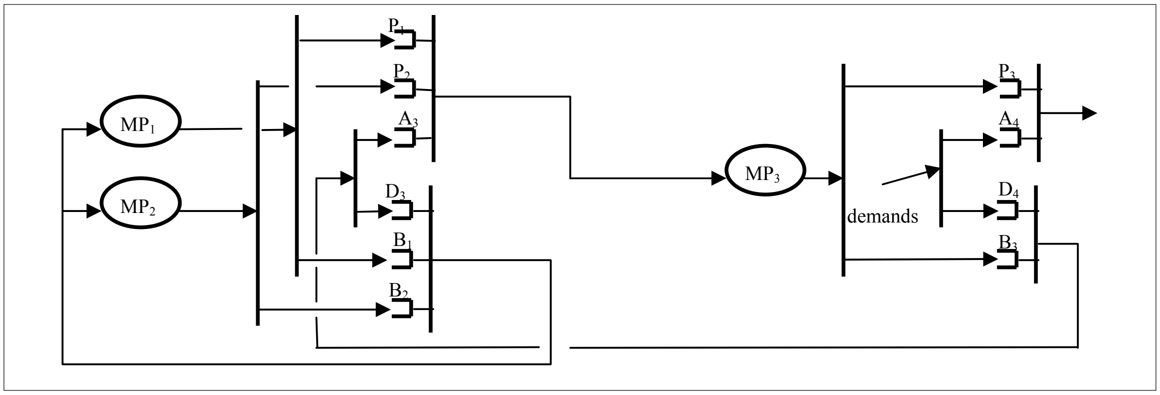

Figure-2. GKS Model.

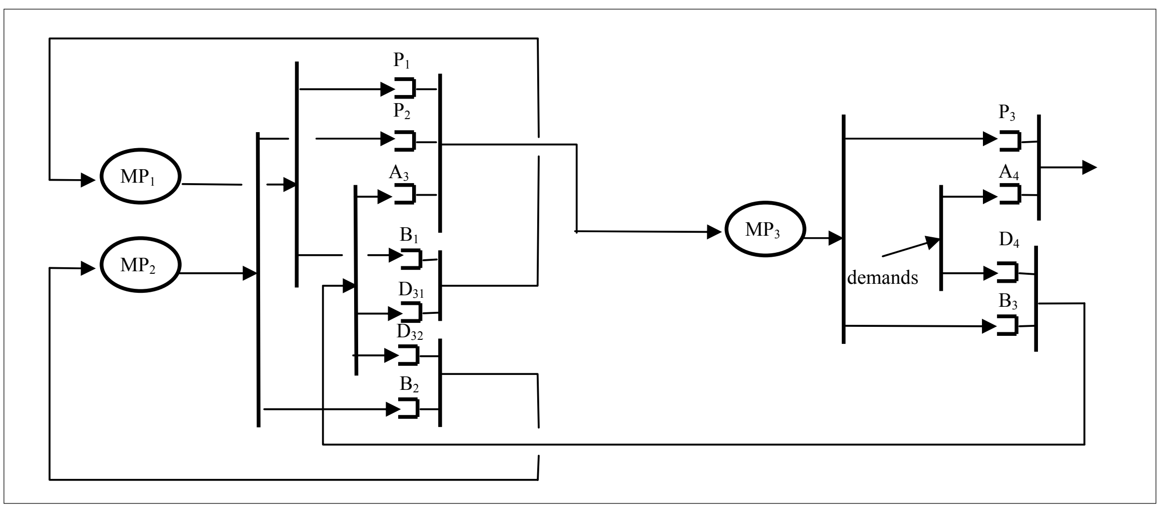

Figure-3. GKI model

<u>In the case of the GKS model (figure 2):</u>

- The kanban is then put in queue A_3 and authorizes the simultaneous release of one stage-1 finished part from P_1 and one stage-2 finished part from P_2 (with the kanban which is in queue A_3, into MP_3 (if P_1 and P_2 are not empty, otherwise the kanban is backordered in A_3). During the transfer towards stage 3, we assume that the synchronization station composed of queues P_1, P_2 and A_3 realizes the instantaneous assembly of one part from P_1 and one part from P_2.
- The demand is put in queue D_3 and constitutes a simultaneous production demand for stages 1 and 2. The production actually starts in MP_1 et MP_2 if there are available kanbans in queues B_1 and B_2 (if B_1 or B_2 is empty, the production demand is backordered until these queues become both not empty); the stage-1 (respectively stage-2) kanban which is in B_1 (B_2) is attached to the part currently being processed in MP_1 (MP_2) and the demand in queue D_3 is discarded.

In the case of the GKI model (figure 3):

- The kanban is again put in queue A_3 and authorizes the simultaneous release of one stage-1 finished part from P_1 and one stage-2 finished part from P_2. The synchronization station composed of A_3, P_1, and P_2, realizes the instantaneous assembly of one stage-1 finished part from P_1 and one stage-2 finished part from P_2, then the assembled product and its attached kanban are released into MP_3.
- The demand divides into two entities, which supply simultaneously queues D_{31} and D_{32}. The entity in D_{31} (respectively D_{32}) constitutes a production demand for stage 1 (stage 2). The production starts actually in MP_1 (MP_2) if there is at least one available kanban in B_1 (B_2), otherwise, the demand is backordered until B_1 (B_2) is supplied ; the stage-1 kanban (respectively stage-2 kanban) is then attached to the part currently being processed in MP_1 (MP_2) and the entity in D_{31} (D_{32}) is discarded. We have to notice that when a production demand for stages 1 and 2 is backordered in queues D_{31} and D_{32}, the effective production in these two stages is independently performed.

III. INVARIANCE PROPERTIES

There are several closed sub-networks within the two proposed models (for example $\{MP_1, B_1\}$ or $\{A_3, MP_3, B_3\}$). The number of customers present in these closed sub-networks is thus constant and equal to the number of customers that are present at the initial state. For each of the GKS and the GKI systems, we can then define the invariants, corresponding to these sub-networks, and thus, some properties [SBI 2000]. Note that, in the remainder of the paper, the content of a given queue is denoted by a symbol written in bold (for example, we write $\mathbf{MP_i}$, to denote the number of parts in queue MP_i)

III.1 GKS model properties

The first two following inequalities, determine, for each stage of the system and at any time, the maximal work-in-process in the stage and the maximal number of available kanbans in this stage.

The last relation provides the maximal number of release authorizations of semi-finished products from stages 1 and 2, towards stage 3.

$\mathbf{MP_i} \le K_i$ (1) $\quad \mathbf{B_i} \le K_i$ (2) $\quad i = 1, 2, 3$

$\mathbf{A_3} \le K_3$ (3)

The following relations provide, for stage i (i= 1, 2, 3), the maximal number of stage-i finished parts, and the maximal number of entities present in the set constituted by queues P_i et MP_i.

$\mathbf{P_i} \le S_i + K_3$ (4) $\quad \mathbf{MP_i} + \mathbf{P_i} \le S_i + K_3$ (5) $\quad i = 1, 2$

$\mathbf{P_3} \le S_3$ (6) $\quad \mathbf{MP_3} + \mathbf{P_3} \le \max(S_3 , K_3)$ (7)

Inequality (8) defines a minimal bound for the number of available kanbans at a given time in stage i (i = 1, 2). We can deduce that it is not necessary to put more than $S_i + K_3$ kanbans in B_i at the initial state, otherwise they will not be utilized.

$$\mathbf{B_i} \ge K_i - S_i - K_3 \qquad i = 1, 2 \qquad (8)$$

Relation (9) determines the maximal number of customers in queue D_3, which contains production demands (sent out by stage 3), for stages 1 and 2.

$$\mathbf{D_3} \le \min (S_1 + K_3 , S_2 + K_3) \qquad (9)$$

III.2 GKI model properties

In this case, inequalities (1) to (8) remain true for the GKI model of figure 3. Moreover we establish the following relation,

$$\mathbf{D_{3i}} \le S_i + K_3 \qquad i = 1, 2 \qquad (10)$$

It is important to notice that for both proposed assembly mechanisms, the obtained models do not conserve the particular characteristics of the Generalized Kanban mechanisms; indeed, the contents of the components buffers can now exceed their initial value.

The queueing networks of figures 2 and 3 model Generalized Kanban controlled assembly systems, realising an assembly operation between two components. This modelling and the resulting properties can easily be generalized to the assembly of any number of components, say L.

IV. TIME EVOLUTION OF GKS AND GKI MODELS

Our purpose here is to determine the basic equations that describe the evolution of GKS and GKI systems. Once the dynamics of these two systems will be defined, we will be able to analyze the influence of the parameters on the departure times of parts from different points of the system. We will also compare the evolution of the two systems relatively to some criteria.

For the two considered models, let us introduce the following notations relative to the occurrence time of some events:

$I_{i,n}$ is the time of the n^{th} arrival in MP_i $\quad i = 1, 2, ..., L+1$

$I_{L+2,n}$ is the time of the n^{th} departure from the system.

$O_{i,n}$ is the time of the n^{th} departure from MP_i $\quad i = 1, 2, ..., L+1$

D_n is the arrival time of the n^{th} demand.

$\sigma_{i,n}$ is the processing time of the n^{th} part on MP_i $\quad i = 1, 2, ..., L+1$

We have, for n, m $\in$ {1, 2, …}, the following inequalities:

$$I_{i,n-m} \le I_{i,n} \qquad i = 1, 2, ..., L+2 \qquad (11)$$
$$O_{i,n-m} \le O_{i,n} \qquad i = 1, 2, ..., L+1 \qquad (12)$$
$$D_{n-m} \le D_n \qquad (13)$$

We assume that if $n \le 0$, then:

$$I_{1,n} = I_{2,n} = .. = I_{L+2,n} = O_{1,n} = O_{2,n} = . . = O_{L+1,n} = D_n = 0$$

IV.1 Evolution equations in the case of GKS systems

For GKS systems, the particular event times we defined above, are related by the following evolution equations (proofs are given in [SBI 2000]):

$$I_{1,n} = \max (D_n , O_{1,n-K_1} , ... , O_{L+1,n-K_{L+1}}) \qquad (14)$$
$$I_{L,n} = ... I_{2,n} = I_{1,n} \qquad (15)$$
$$I_{L+1,n} = \max (D_n , O_{1,n-S1} , ..., O_{L,n-S_L} , O_{L+1,n-K_{L+1}}) \qquad (16)$$
$$I_{L+2,n} = \max (D_n , O_{L+1,n-S_{L+1}}) \qquad (17)$$
$$O_{i,n} = \sigma_{i,n} + \max (I_{i,n} , O_{i,n-1}) \qquad i = 1, 2, ..., L+1 \qquad (18)$$

IV.2. Evolution equations in the case of GKI systems

With the same notation, we obtain similar equations for assembly systems with independent release:

$$I_{i,n} = \max (D_n , O_{i,n-K_i} , O_{L+1,n-K_{L+1}}) \qquad i = 1, 2, ..., L \qquad (19)$$
$$I_{L+1,n} = \max (D_n , O_{1,n-S_1}, ..., O_{L,n-S_L} , O_{L+1,n-K_{L+1}}) \qquad (20)$$
$$I_{L+2,n} = \max (D_n , O_{L+1,n-S_{L+1}}) \qquad (21)$$
$$O_{i,n} = \sigma_{i,n} + \max (I_{i,n} , O_{i,n-1}) \; i = 1, 2, ..., L+1 \qquad (22)$$

IV.3. Comparison of GKS and GKI systems evolution

Intuitively, a visual comparison between the GKS model of figure 2 and the GKI model of figure 3 may lead to say that the GKI system responds faster to an external demand than a GKS system with the same parameters values. This is due to the relative independence between the manufacturing processes which provide the components in the case of a GKI system. That is what we want to prove in this section.

The 'S' exponent is attributed to the entities relative to the GKS system and the 'I' exponent to those relative to the GKI system. Then we have:

Property 1:

The two following relations enable to compare between the GKS and the GKI systems particular event times.

$$I_{i,n}^{\ I} \le I_{i,n}^{\ S} \qquad i = 1, 2, ..., L+2 \qquad (23)$$
$$O_{i,n}^{\ I} \le O_{i,n}^{\ S} \qquad i = 1, 2, ..., L+1 \qquad (24)$$

Proof:

- At the initial state (n = 0), the above inequalities are true, because in this case, all the considered entities are equal to zero.
- Then we assume (Hypothesis $\mathbf{H_1}$) that all the above inequalities hold up to n-1, that is :

Hypothesis $\mathbf{H_1}$: $I_{i,n-m}{}^{I} \le I_{i,n-m}{}^{S}$ $i = 1, 2, ..., L+2$

$O_{i,n-m}{}^{I} \le O_{i,n-m}{}^{S}$ $i = 1, 2, ..., L+1$

for m = 1 , 2 , …

- Since there exists a total ordering on the event times, since inequalities (23) and (24) hold for n = 0, and assuming that $\mathbf{H_1}$ is verified, if we prove that these inequalities hold for n, then they will hold for any value of n.

1) First, let us prove inequality (23) for i = 1, 2, ..., L. From equations (14), (15) and (19), we have:

$I_{i,n}{}^{S} = \max (D_n{}^{S}, O_{1,n-K1}{}^{S}, \ldots , O_{L+1,n-K_{L+1}}{}^{S})$ $i = 1, 2, ..., L$

$I_{i,n}{}^{I} = \max (D_n{}^{I}, O_{i,n-Ki}{}^{I}, O_{L+1,n-K_{L+1}}{}^{I})$ $i = 1, 2, ..., L$

In order to compare these two event times, we assume that $D_n{}^{S} = D_n{}^{I}$ (the n^{th} demand arrives at the same time in the two systems). From $\mathbf{H_1}$, we have:

$O_{i,n-Ki}{}^{I} \le O_{i,n-Ki}{}^{S}$ $i = 1, 2, ..., L$

$O_{L+1,n-K_{L+1}}{}^{I} \le O_{L+1,n-K_{L+1}}{}^{S}$

And then, for any value of $O_{j,n-Kj}{}^{S}$ ($j \in \{1, 2, ..., L\} - \{i\}$), relation (23) holds for i = 1, 2, ..., L.

2) To prove inequality (23) for i = L+1, we can write, according to relations (16) and (20):

$I_{L+1,n}{}^{S} = \max (D_n , O_{1,n-S1}{}^{S}, .., O_{L,n-S_L}{}^{S}, O_{L+1,n-K_{L+1}}{}^{S})$

$I_{L+1,n}{}^{I} = \max (D_n , O_{1,n-S1}{}^{I}, .., O_{L,n-S_L}{}^{I}, O_{L+1,n-K_{L+1}}{}^{I})$

And from $\mathbf{H_1}$, we have:

$O_{i,n-S_i}{}^{I} \le O_{i,n-S_i}{}^{S}$ $i = 1, 2, ..., L$

$O_{L+1,n-K_{L+1}}{}^{I} \le O_{L+1,n-K_{L+1}}{}^{S}$

Then: $I_{L+1,n}{}^{I} \le I_{L+1,n}{}^{S}$

3) Let us prove inequality (23) for i = L+2. From relations (17) and (21), we have,

$I_{L+2,n}{}^{S} = \max (D_n , O_{L+1,n-S_{L+1}}{}^{S})$

$I_{L+2,n}{}^{I} = \max (D_n , O_{L+1,n-S_{L+1}}{}^{I})$

And from $\mathbf{H_1}$, we have:

$O_{L+1,n-S_{L+1}}{}^{I} \le O_{L+1,n-S_{L+1}}{}^{S}$

Then $I_{L+2,n}{}^{I} \le I_{L+2,n}{}^{S}$.

4) Let us prove inequality (24). From inequalities (18) and (22), we have:

$O_{i,n}{}^{S} = \sigma_{i,n}{}^{S} + \max (I_{i,n}{}^{S}, O_{i,n-1}{}^{S})$ $i = 1, 2, ..., L+1$

$O_{i,n}{}^{I} = \sigma_{i,n}{}^{I} + \max (I_{i,n}{}^{I}, O_{i,n-1}{}^{I})$ $i = 1, 2, ..., L+1$

In order to compare these two event times, we assume that MP_i has the same processing time in the two systems, that is:

$\sigma_{i,n}{}^{S} = \sigma_{i,n}{}^{I}$

From relation (23), we have:

$I_{i,n}{}^{I} \le I_{i,n}{}^{S}$ $i = 1, 2, ..., L+1$

And from $\mathbf{H_1}$, we have:

$O_{i,n-1}{}^{I} \le O_{i,n-1}{}^{S}$

Then we obtain (24).

♦

This comparison allows us to conclude that GKI systems respond to a consumer's demand as fast as or faster than GKS systems.

IV.4. Influence of the parameters on the behavior of the system

In this section we are interested in the influence of the parameters on the evolution of each of the GKS and GKI systems. We use the same notations as in the previous sections, and we add a (') when a parameter is modified (we change the value of a single parameter at the same time). Let us recall that the parameters, for the two proposed models, are K_i and S_i, i = 1, 2, ..., L+1.

Moreover inequalities (11) and (12) established before, let us present some obvious relations, but necessary for proving some important properties:

$I'_{i,n-m} \le I'_{i,n}$ $i = 1, 2, ..., L+2$ (25)

$O'_{i,n-m} \le O'_{i,n}$ $i = 1, 2, ..., L+1$ (26)

for m = 1 , 2 , …

However, it is clear that the arrival times of demands do not depend on the system parameters; so we have $D'_n = D_n$.

IV.4.1 *Influence of the parameters in the case of GKS systems*

Let us study the effect of the modification of one parameter on the GKS systems behavior. We have the following property:

Property 2:
If the value of one of the parameters of a GKS system is replaced by a strictly superior value and if all the other parameters remain unchanged, we have:

$$I'_{i,n} \leq I_{i,n} \qquad i = 1, 2, ..., L+2 \qquad (27)$$

$$O'_{i,n} \leq O_{i,n} \qquad i = 1, 2, ..., L+1 \qquad (28)$$

Proof:
Let us show that this property is true if the modified parameter is K_1. The proof of property 2 is similar when one of the other parameters is modified into a strictly superior value.

- At the initial state (n = 0), the above inequalities are true, because in this case, all the considered entities are equal to zero.
- Then we assume (hypothesis $\mathbf{H_2}$) that all the above inequalities hold up to n-1, that is :

Hypothesis $\mathbf{H_2}$:

$$I'_{i,n-m} \leq I_{i,n-m} \qquad i = 1, 2, ..., L+2$$

$$O'_{i,n-m} \leq O_{i,n-m} \qquad i = 1, 2, ..., L+1$$

for m = 1 , 2 , …

1) To prove (27) for i = 1, 2, ..., L (the proof for i = L+1 and i = L+2 is similar), from equations (14), (15) and (19), we have:

$$I'_{i,n} = \max (D'_n , O'_{1,n-K'_1} , O'_{2, n-K'_2} , ..., O'_{L+1,n-K'_{L+1}})$$

And from equation (26), since $n-K'_1 < n- K_1$, then:

$$O'_{1,n-K'_1} \leq O'_{1,n-K_1}$$

Moreover, we have:

$$D'_n = D_n$$

$$K'_i = K_i \qquad i = 2, ..., L+1$$

Then: $I'_{i,n} \leq \max (D_n , O'_{1,n-K_1} , O'_{2,n-K_2} , ..., O'_{L+1,n-K_{L+1}})$

And from $\mathbf{H_2}$,

$$O'_{i,n-K_i} \leq O_{i,n-K_i} \quad \text{for} \quad i = 1, 2, ..., L+1.$$

Then: $I'_{i,n} \leq \max (D_n , O_{1,n-K_1} , O_{2,n-K_2} , ..., O_{L+1,n-K_{L+1}}) = I_{i,n}$

2) Now let us prove relation (28). We have:

$$O'_{i,n} = \sigma'_{i,n} + \max (I'_{i,n} , O'_{i,n-1}) \qquad i = 1, 2, ..., L+1$$

The processing times in the different manufacturing processes are independent from the system parameters, so we have:

$$\sigma'_{i,n} = \sigma_{i,n}$$

From $\mathbf{H_2}$, and since $O'_{i,n-1} \leq O_{i,n-1}$

Then we have: $O'_{i,n} \leq \sigma_{i,n} + \max (I'_{i,n} , O_{i,n-1}) \qquad i = 1, 2, ..., L+1$

Finally, from (27), $O'_{i,n}$ is then bounded by $O_{i,n}$.

♦

Consequently, if we increase the value of one parameter in a GKS system, it generally responds faster to a consumer's demand.

IV.4.2. *Influence of the parameters in the case of GKI systems*

Now let us study the effect of the modification of one parameter on the behavior of the GKI system.We have the following property:

Property 3:
If the value of one of the parameters of a GKI system is replaced by a strictly superior value and if all the other parameters remain unchanged, we have:

$$I'_{i,n} \leq I_{i,n} \qquad i = 1, 2, ..., L+2 \qquad (29)$$

$$O'_{i,n} \leq O_{i,n} \qquad i = 1, 2, ..., L+1 \qquad (30)$$

The proof for property 3 is similar to the one of property 2.

Consequently, if we increase the value of one parameter in a GKI system, it generally responds faster to a consumer's demand.

In conclusion, for a Generalized Kanban policy, and for both assembly strategies (with simultaneous or independent release), when the value of one parameter is increased, the response time to a consumer's demand, becomes generally shorter.

V. CONCLUSION

The problem of the modeling of Kanban controlled assembly systems has already been tackled and queueing networks models have been proposed in [DI 1996].

In this paper, we have generalized these results to the Generalized Kanban controlled assembly systems, which include the Kanban controlled assembly systems and the Base-Stock controlled assembly systems as particular cases.

The results we established show that, unlike in the case of classical Kanban controlled assembly systems, the obtained queueing network models do not conserve the special characteristics of the Generalized Kanban systems composed by serial stages. We notice that the stocks of components are not bounded by their initial contents anymore; moreover they can exceed them, and can theoretically reach infinite values, in the particular case of the Base-Stock policy.

On the other hand, when we compare the two established models, we notice that the GKI system responds as fast as or faster than the GKS system, to an external demand; in fact this depends on the distribution of times between demands arrivals, and on the distribution of the service times of the considered manufacturing processes.

In the same way, when the value of one parameter is increased in a GKS or a GKI system, the response time remains unchanged or becomes smaller; also in this case, this depends on the distribution of times between demands arrivals, and on the distribution of the service times in the considered manufacturing processes.

The models we proposed for Generalized Kanban controlled assembly systems, let us envisage the analysis of these systems, in the purpose of their performance evaluation.

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