REVIEW OF SOME QUEUEING MODELS FOR MANAGING INVENTORIES, BACKORDERS, AND QUALITY JOINTLY IN STOCHASTIC MANUFACTURING SYSTEMS

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Abstract: This paper reviews some recent applications of queueing theory and convex analysis in determining customer admission, inventory, and quality control policies jointly in manufacturing systems that produce a single product to meet random demand. These problems have been addressed separately in the past. We consider admission and production control policies of the threshold type whereby the system produces until stock reaches a certain level and accepts orders until its backlog reaches another critical level. The problem is to find the optimal quality tolerances and the critical stock and backlog levels which maximize the mean profit rate of the system. We also examine a supply chain comprising two factories in series with external subcontractors, and we compare the benefit of determining the optimal base stocks for the two systems jointly rather than individually. From theoretical and numerical results, it appears that managing inventory levels, sales, and quality tolerances jointly achieves higher profit than independently determined control policies.

Keywords: Queueing networks, admission control, inventory control, quality control, global optimization.

1. Introduction

In this paper, we consider two optimization problems for manufacturing systems that produce a single product to meet random demand. Our aim is to show the benefits of coordination among the various departments of a manufacturing system as well as among different firms in a supply chain.

First, we consider a production network in which all outgoing items are inspected for some quality characteristic. Each product is classified as conforming, if the quality characteristic falls inside some interval around a certain target value; otherwise it is classified as nonconforming. We address the following three questions: when to produce, when to accept an incoming order, and which quality control procedure to adopt. The aim is to maximize net profit, that is, revenue from sales less purchase, production, inventory carrying, backlog, and quality costs.

Second, we examine a supply chain of two manufacturers that produces a single product type. The first manufacturer provides the basic component of the final product, and the second one makes the final product. Each manufacturer maintains a stock of finished items. During a stockout period, the demand for finished products or components is satisfied immediately from subcontractors, at a cost that reflects the corresponding loss of profit for each firm. Again, the aim here is to control the production in each system so as to balance inventory and subcontracting costs.

A practical approach to solve problems of inventory control is to search in a class of simple policies that depend on a small number of parameters. A common inventory control policy for single-machine systems is one that specifies a target value for the number of finished items, called the *base stock*. When the buffer level reaches the base stock, the machine is switched off. This policy ensures that the inventory cost is bounded. Systems operating under base stock policies and some extensions including Kanban control, have been studied extensively in the past (see, e.g., [1]). Although base stock policies are not optimal for multistage production systems, their performance is often satisfactory [2] and they are easily implementable. For example, [3] and [4] use queueing theory to determine the operating characteristics of production networks and the base stock levels that maximize a given performance measure.

If arriving orders cannot be filled immediately from stock, they are usually unconditionally either rejected [*lost sales policy* (LS)] or accepted [*complete backordering policy* (CB)] [5]–[7]. A more general admission policy is one that rejects orders when backlog reaches a certain limit, called the *base backlog*, and accepts them otherwise. This partial backordering or *partly lost sales policy* (PLS), as we shall call it hereafter, was proposed in [8] for the problem of controlling the entry to an $M/M/1$ queueing system. Single-stage production systems with base stock control and partly lost sales have been studied in [9]. When the base backlog is zero, PLS becomes LS, and when the base backlog is infinite, PLS becomes CB.

Finally, a common quality control practice is the design of complete inspection plans, also known as screening or 100%-inspection procedures. In a typical screening procedure, each outgoing item is inspected for some quality characteristic. The item is classified as nonconforming, if its quality characteristic falls outside the specification region, which is a closed interval around a certain target value; otherwise it is classified as conforming. A detailed literature review on the design of screening procedures can be found in [10].

In this paper, we outline some recent results on managing inventories, order admissions, and product quality jointly for a production network and on managing inventories jointly in a supply chain with two factories. These problems are solved using basic queueing theory and exhaustive search supported by convex analysis. From numerical results, it appears that the jointly optimal control designs achieve higher profit than locally optimal control actions.

2. A Make-to-Stock Production Network with Partly Lost Sales

2.1 Description and Equivalent Queueing Network

We consider a manufacturing system of the type shown in Fig. 1. The system consists of a production facility with several machines and intermediate buffers and three additional buffers (depicted by large circles): an input buffer for storing raw parts, an output buffer for finished items and one more which keeps track of outstanding orders.

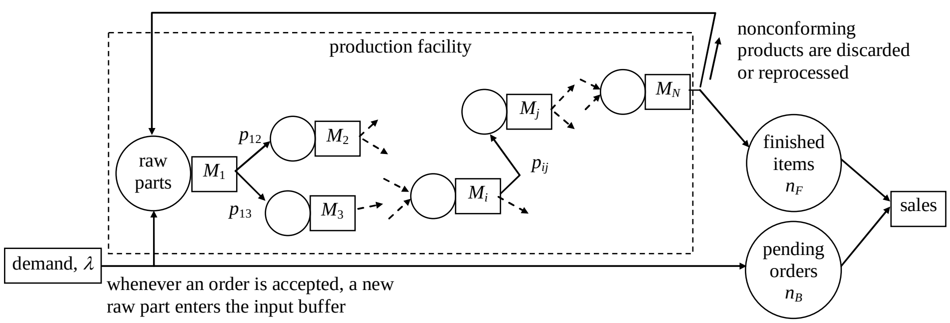

Figure 1**.** Production system.

The times between successive customer arrivals are independent, exponentially distributed random variables with mean $1/\lambda$ and each customer requests one unit of product. We assume that the production facility is a queueing network of the Jackson type with N nodes $i = 1, 2, \ldots, N$. Node i comprises a buffer and a machine M_i which is fed by the buffer. Parts leaving machine M_i are sent to the buffer of machine M_j for the next operation with probability p_{ij}. The processing times of machine M_i are independent, exponential random variables with mean $1/\mu_i$. Machine M_1 performs the first operation and machine M_N produces the end products. Each outgoing product is inspected and classified as conforming or nonconforming depending on the value Y of some quality characteristic. If Y lies within a specified acceptance region $[t-\delta, t+\delta]$ around a target value t then the product is declared as conforming and ready for sale; otherwise the product is discarded or reworked. We assume that the probability density function of Y is known.

The control of sales and stock is accomplished according to the following rules: (a) At time zero, the inventory position of the system is fixed at s, where s is a nonnegative integer which will be referred to as the *base stock*. The inventory position is the total number of parts in the system less the number of pending orders. (b) When a customer arrives the corresponding order is accepted or denied depending on the current number of pending orders. If c orders are pending the new order is either rejected or satisfied by subcontractors. We shall refer to c as the *base backlog*. When an order is accepted a new raw part is placed in the input buffer. Thus the inventory position of the system is unaltered. (c) A sale occurs instantly whenever there is one finished item and one pending order in the system. Thus, the number of products and the number of pending orders cannot be both positive simultaneously. Again, the inventory position of the system is not affected by sales.

Suppose that, at some time instant, the number of items in node i (buffer plus machine M_i) is n_i, $i = 1, 2,\ldots, N$, the number of finished items is n_F and the number of pending orders is n_B. By the memoryless property of the exponential distribution, the evolution of the system is independent of past states given the current state (n_F, n_B; n_i, $i = 1,\ldots,N$) and it is described by a Markov chain. The total number n_H of items in the system is given by $n_H = n_F + n_1 +\ldots+n_N$. It follows from rules (a)–(c) above that the inventory position $n_H - n_B$ is always s. This gives rise to an alternative expression for n_H, $n_H = s + n_B$.

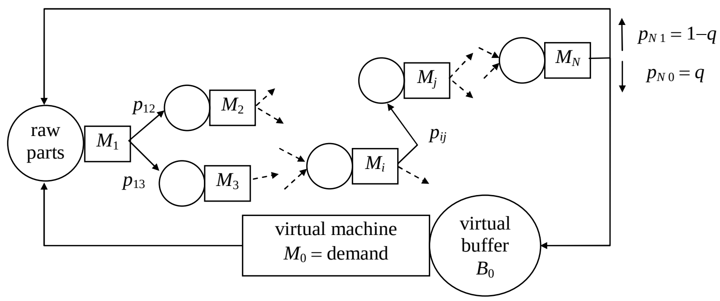

Figure 2. Equivalent closed queueing network

Let q be the probability that a part moving out of M_N is conforming, that is, $q = P(t-\delta \leq Y \leq t+\delta)$. Then, system shown in Fig. 1 is equivalent to the closed queueing network (CQN) of Fig. 2, which is obtained by reversing the flow of pending orders. This CQN has a constant population of $m = s+c$ parts and N+1 nodes. The nodes $i = 1, 2, \ldots, N$ have the same buffer contents n_i and processing rates μ_i as those in the production facility of the original system and there is an extra node, 0, which consists of a virtual machine M_0 and a virtual buffer B_0. The processing times of M_0 are the same as the interarrival times in the original system. Hence, $\mu_0 = \lambda$. It then remains to express n_F and n_B, and n_H of the original system in terms of the level n_0 of the virtual buffer B_0 in the equivalent system. Table I summarizes this relationship. A general method for simplifying queueing networks by reversing flows has been proposed in [11].

Table I. Relationship between the original system and the CQN

n_F	n_B	n_H	n_0
0	0	s	c
1	0	s	c+1
…	…	…	…
s	0	s	$s+c = m$
0	1	s+1	c–1
…	…	…	…
0	c	$s+c = m$	0

2.2 Performance Measure

The overall performance measure of the system is the mean profit rate. This quantity depends on the sales (system throughput) and the costs of inventory, backlog (delays in filling orders), and quality. We consider six profit or cost parameters:

- p unit profit (selling price less cost of purchasing raw parts and processing per item),
- h unit holding cost rate (cost per unit time per item in the system),
- b unit backlog cost rate (cost per unit time of delay for a pending order),
- i_C inspection cost per outgoing item,
- r_C rework/rejection cost per nonconforming item, and
- k a quality loss parameter.

The quality loss parameter is associated with a cost due to the deviation of Y from its target t. This cost is called the quality loss. A common quality loss function is the quadratic function $k(Y-t)^2$, which has been proposed by Taguchi [12]. Let q be the probability of producing a conforming item, that is, $q = P(t-\delta \le Y \le t+\delta)$. Then the mean quality cost per outgoing item is given by

$$Q = i_C + (1-q)r_C + qk\, E\,[(Y-t)^2 |\, t-\delta \le Y \le t+\delta]$$

where $E[.]$ is the expectation operator. The quantities q and $E\,[(Y-t)^2 |\, t-\delta \le Y \le t+\delta]$ can be computed from the probability density function of Y. Let TH be the throughput of the system, that is, the mean rate of conforming items. Then total rate of outgoing products (conforming or nonconforming) is TH/q and the mean total quality cost rate is Q TH/q. Finally, the mean profit rate of the system is given by

$$J(\delta, m, s) = p\mathrm{TH} - hH - bB - Q\ \mathrm{TH}/q$$

where δ, m, and s are the control parameters, H is the average inventory and B the average backlog of the system.

Next, we derive expressions for TH, H, and B using the routing probabilities p_{ij} and the rates μ_i of the machines. Let Π denote the matrix of routing probabilities, $\Pi = [p_{ij}]$, and $U = [U_0\ U_1\ \ldots\ U_N\ ]$ *any nonnegative solution* of the system of linear equations $U = U\Pi$. For example, we may set $U_0 = \lambda$ and solve the system of equations for the remaining entries of U. Then, in equilibrium, the probability that the equivalent system will be in state $(n_0, \ldots, n_N)$ is given by [13]

$$P(n_0, \ldots, n_N) = \frac{1}{G(m)} \prod_{i=0}^{N} \left[\frac{U_i^{n_i}}{\mu_i(1) \ldots \mu_i(n_i)} \right]$$

where $G(m)$ is a normalizing constant defined so that all the probabilities sum to one and $\mu_i(n)$ is the mean production rate of node i when $n > 0$ customers are present at that node, otherwise $\mu_i(0) \triangleq 1$. For the production network studied here we have $\mu_i(n) = \mu_i$. The throughput of the system is the mean rate of accepted orders. Hence, TH $= \mu_0\ P(n_0 > 0)$. Also, the average inventory H and backlog B are computed from

$$H \triangleq E\,[n_H\,] = sP(n_0 \ge c) + (s+1)P(n_0 = c-1) + \ldots + mP(n_0 = 0)$$
$$B \triangleq E\,[n_B\,] = 1P(n_0 = c-1) + 2P(n_0 = c-2) + \ldots + cP(n_0 = 0)$$

2.3 Optimization

Having obtained explicit expressions for the various cost parameters, we now consider the problem of maximizing the mean profit rate of the system $J(\delta, m, s)$. Since the optimization problem is three-dimensional, we solve it sequentially. (a) First, we maximize $J(\delta, m, s)$ with respect to s for any fixed δ and m. (b) Then, we maximize the profit with respect to m for any fixed δ. Finally, we perform exhaustive search by repeating steps (a) and (b) for different values of δ. The following theorems [14] facilitate the search for the optimal values of s and m in steps (a) and (b).

Theorem 1: (a) The function $J(\delta, m, s)$ is concave in s for any fixed m and δ and assumes its maximum value at the point $s_{\delta,\, m}$ which satisfies the following condition

$$\frac{G(s_{\delta,m} - 1)}{G(m)} \le \frac{b}{h+b} < \frac{G(s_{\delta,m})}{G(m)}$$

(b) Furthermore, $s_{\delta,\, m+1} \in \{s_{\delta,\, m},\ s_{\delta,\, m}+1\}$.

Theorem 2: The function $J(\delta, m, s_{\delta,\, m})$ is quasiconcave in m for any fixed δ and for $m \in \{M, M+1, \ldots\}$, where M is the smallest solution of the inequality

$$h \ge p\mu_0 \left[\frac{G_0(m+1)}{G_0(m+2)} - \frac{G_0(m)}{G_0(m+1)} \right] \tag{1}$$

and $G_0(.)$ is the normalization constant of the CQN of Fig. 2 with node 0 removed, $p_{N0} = 0$, and $p_{N1} = 1$.

The above theorems hold for production networks of the Jackson type that incur a constant inventory cost rate h throughout the manufacturing process. We now weaken these assumptions as follows:

(1) The system employs rules (a)–(c) of Section 2.1 to determine when an incoming order is accepted and when a new raw part is released into the system. (2) The times between successive requests for products are random variables with general distributions, independent of the state of the manufacturing system. The processing times and the routing of parts may depend on the states of the machines and the buffer levels of the production facility. (3) The system incurs holding cost $f(n_1, n_2, \ldots, n_N)$ per time unit when n_i parts are in machine i for all $i = 1, 2, \ldots, N$, and a holding cost hn_F when n_F items are in the output buffer. (4) For any fixed s and c, the system is ergodic in the sense that its average production rate and the average holding and backlog cost rates during a period $[0, t]$ converge to the mean values TH, $E[f(n_1, \ldots, n_N)]$, $hE(n_F)$, and $bE(n_B)$ as $t \to \infty$, for every initial state with probability 1.

The following theorem [14] is a partial extension of Theorem 1 to more general production systems.

Theorem 3: Under assumptions (1)–(4), the function $J(\delta, m, s)$ is concave in s for any fixed m and δ.

Finally, we have verified the validity of Theorems 1 and 2 for systems with a single-stage unreliable production facility and intermittent demand. For such systems, condition (1) holds for every $m \geq 0$.

2.4 Numerical Results

In this section, we compare the profits achieved by full coordination between the production and quality control departments of a production system and by two other practices in which there is partial or no coordination between these departments. The three optimization strategies are as follows: (1) FULL: This strategy seeks values for δ, m, and s that jointly maximize the mean profit rate of the system. (2) PARTIAL: Here the quality control department computes the value $\delta = \sqrt{r_c/k}$, which minimizes the mean quality cost Q per outgoing item [10]. Using this value, the production department computes the probability q of a conforming item. Then, m and s are determined so as to maximize the quantity $p\mathrm{TH} - hH - bB$ which is the total profit without quality costs. (3) NO (no coordination): The quality control department determines δ from the above equation but the production department ignores this information assuming that $\delta = \infty$ or, equivalently, $q = 1$ and computes the values for m and s that maximize the total profit without quality costs.

We combine these strategies with the partly lost sales policy (PLS), the complete backordering policy (CB), and the lost sales policy (LS).

We consider a manufacturing system whose production facility is a production line with six machines similar to the one examined in the previous section. The parameters of the system are: $\lambda = 4.0$, $\mu_1 = 6.0$, $\mu_2 = 7.0$, $\mu_3 = 5.0$, $\mu_4 = 5.5$ $\mu_5 = 6.5$, $\mu_6 = 5.0$, $t = 9.2$, $p = 100.0$, $h = 5.0$, $b = 6.0$, $k = 25.0$, $r_C = 20.0$, $i_C = 1.0$, and Y is normally distributed with mean $\overline{y} = 10.0$ and variance $\sigma^2 = 0.5$. We vary the parameters λ, considering all combinations of admission policies (PLS, CB, LS) and optimization strategies (FULL, PARTIAL, NO).

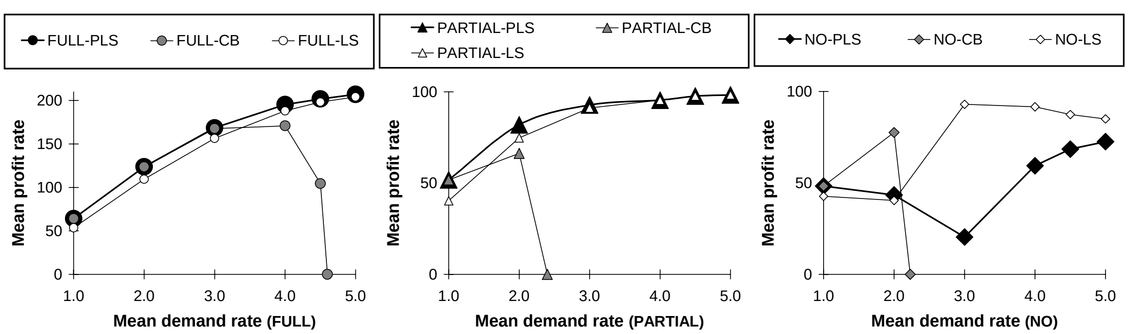

Figure 3. Performance of FULL, PARTIAL, and NO versus λ.

The results are shown in Fig. 3. Missing combinations designate that the corresponding policy achieves negative profit rate. When the demand rate is low PLS and CB perform alike and their profits are higher than those of LS by 20%. As λ increases the production facility begins to operate at full capacity (nodes 3 and 6 are the bottlenecks of the facility having a capacity of 5 parts per time unit). In this case the performance of CB incurs a severe degradation because the backlog increases without bound. Finally, when the demand rate exceeds the capacity of the production facility PLS and LS perform alike. In all cases, the proposed admission policy (PLS) achieves superior performance. Under this policy, when the demand rate is low strategies PARTIAL and NO achieve the same profit as that of FULL. However, as λ increases their profit is less than half the profit of FULL.

From the above and several other experiments not reported here due to space limitations, we see that coordination of the production control and the quality control departments is the key to profitability, especially when the demand rate λ is high or when parameters h, b, σ^2, and k are large. Also, the proposed partly lost sales policy outperforms CB and LS in all cases.

3. Coordinated Inventory Control in a Two-Stage Supply Chain

3.1 System Description and Queueing Model

Consider a supply chain of two manufacturers M_1 and M_2, as shown in Fig. 4. The first manufacturer provides the basic component of the final product, and the second one makes the final product. Each manufacturer maintains a stock of finished items. During a stockout period, the demand for finished products or components is satisfied immediately from subcontractors, at a cost that reflects the corresponding loss of profit for each firm. The inventory control in each factory is assumed of the base-stock type, whereby factory M_i produces parts as long as the stock in B_i is lower than a specified level b_i and stops otherwise. The aim here is to specify b_1 and b_2 so as to balance inventory costs and lost sales optimally.

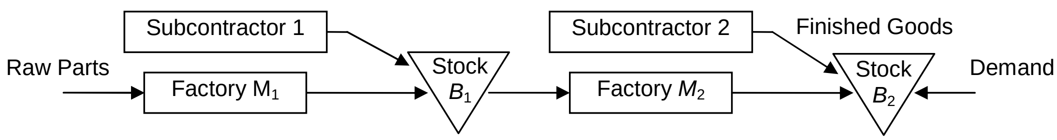

Figure 4. A two-stage supply chain.

Suppose that the processing times of factories M_i, $i = 1, 2$, are exponentially distributed with mean values $1/\mu_i$, and that customers arrive according to a Poisson process with rate λ. More general Markovian distributions can be incorporated into the model without difficulty.

The state of the system is described by the pair (n_1, n_2), where n_i is the stock in B_i and $n_i = 0, 1, \ldots, b_i$. Note that when $n_1 = 0$, M_1 can still provide parts to M_2 (if M_2 is not stopped, i.e., $n_2 < b_2$) by purchasing from subcontractor 1. Similarly, during a stockout period of B_2, all demand is satisfied immediately by manufacturer M_2 who purchases parts from subcontractor 2.

Let $P(n_1, n_2)$ be the equilibrium probability of state (n_1, n_2) and define the indicator function $\mathbf{1}_{(x)} = 1$, if condition (x) is true, otherwise $\mathbf{1}_{(x)} = 0$. The equilibrium probabilities can be computed by solving the Chapman–Kolmogorov equations

$$P(n_1, n_2)\,[\mu_1\mathbf{1}_{(n_1 < b_1)} + \mu_2\mathbf{1}_{(n_2 < b_2)} + \lambda\mathbf{1}_{(n_2 > 0)}] = P(n_1-1, n_2)\mu_1\mathbf{1}_{(n_1 > 0)} + P(n_1+1, n_2-1)\mu_2\mathbf{1}_{(n_1 < b_1),\,(n_2 > 0)}$$
$$+ P(0, n_2-1)\mu_2\mathbf{1}_{(n_1 = 0),\,(n_2 > 0)} + P(n_1, n_2+1)\lambda\mathbf{1}_{(0< n_2 < b_2)} \qquad (2)$$

By defining $P(n_2) \triangleq [P(0, n_2)\ P(1, n_2)\ \ldots\ P(b_1, n_2)]$, the above equations can be expressed in matrix form as follows:

$$P(0)A_0 = P(1)C_0 \qquad (3)$$
$$P(n_2)A = P(n_2-1)B + P(n_2+1)C\,,\ \ n_2 = 1, \ldots, b_2-1 \qquad (4)$$
$$P(b_2)A_1 = P(b_2-1)B_1 \qquad (5)$$

where A_0, A_1, A, C_0, C, B_1, and B are matrices of suitable dimensions that describe the transition rates among the states at a given system level n_2, and they are computed according to Eq.(2). Equations (3)–(5) can be solved sequentially (see, e.g., [15]). We start from Eq. (5) and solve for $P(b_2) = P(b_2-1)B_1A_1^{-1}$. Next, we solve Eq. (4) for $P(b_2-1), \ldots, P(1)$ expressing all these vectors in terms of $P(0)$. Finally, we compute $P(0)$ from Eq. (3) and the normalization condition

$$\sum_{n_2=1}^{b_2}\sum_{n_1=1}^{b_1} P(n_1, n_2) = 1$$

3.2 Performance Measures

The overall performance measure of the system is the mean profit rate. This quantity depends on the sales and the costs of inventory, production, and subcontracting. We consider four profit or cost parameters:

- p_1 price at which factory M_1 sells a component to M_2 (produced by M_1 or by subcontractor 1),
- p_2 selling price of the final product (produced by M_2 or by subcontractor 2),
- c_i unit production cost at factory M_i (c_1 includes the cost of purchasing a raw part),
- s_i cost of purchasing one unit from subcontractor i rather than producing it in factory M_i,
- h_i unit holding cost rate in buffer B_i (cost per unit time per item in stock).

Let STH_i be the mean rate at which factory M_i purchases parts from subcontractor i and TH_i the mean rate (throughput) of parts produced at M_i. Then, $\lambda = \mathrm{TH}_2 + \mathrm{STH}_2$, because all external demand is satisfied either from the production facility of M_2 or from subcontractor 2. Similarly, $\mathrm{TH}_2 = \mathrm{TH}_1 + \mathrm{STH}_1$, because one product produced by M_2 requires one basic component which is either produced by M_1 or provided by subcontractor 1.

The quantities TH_i, STH_i, as well as the mean buffer level H_i of buffer B_i, $i = 1, 2$, can easily be computed from the equilibrium probabilities. For example, STH_2 is the mean demand rate during a stockout period of B_2. Hence, $\mathrm{STH}_2 = \lambda P(n_2 = 0)$ and $\mathrm{TH}_2 = \lambda - \mathrm{STH}_2$. Similarly, TH_1 is the mean production rate of factory M_1. Therefore, $\mathrm{TH}_1 = \mu_1 P(n_1 < b_1)$ and $\mathrm{STH}_1 = \mathrm{TH}_2 - \mathrm{TH}_1$. Also,

$$H_1 = \sum_{n_2=1}^{b_2} \sum_{n_1=1}^{b_1} n_1 P(n_1, n_2)\,,\; H_2 = \sum_{n_2=1}^{b_2} \sum_{n_1=1}^{b_1} n_2 P(n_1, n_2)$$

The mean profit rate of the system is given by

$$J(b_1, b_2) = \lambda p_2 - \sum_{i=1}^{2} \mathrm{TH}_i c_i - \sum_{i=1}^{2} \mathrm{STH}_i s_i - \sum_{i=1}^{2} H_i h_i \tag{6}$$

The optimal pair (b_1^*, b_2^*) is found by exhaustive search on the set $\{0, 1, \ldots, C_1\}\times\{0, 1, \ldots, C_2\}$, where C_i are upper search limits for b_i, $i = 1, 2$. This strategy maximizes the overall profit of the system and will be referred to as GLOBAL.

Suppose now, that the two manufacturers make optimal decisions by pursuing only local concerns. Specifically, M_1 uses the optimal base stock b_1 that maximizes its own profit and shares this information with manufacturer M_2, who then finds the locally optimal base stock b_2. We call this strategy LOCAL.

The individual profits are computed as follows. Since there are always components available for M_2, the second factory can be modeled as an $M/M/1/b_2$ queueing system with arrival rate μ_2 and service rate λ. Let $P_1(n_2) = (1 - \rho)\rho^{n2}/(1 - \rho^{1+b2})$ be the equilibrium probability of this queueing system (see, e.g., [8]), where $\rho = \mu_2/\lambda$ and $n_2 = 0, 1, \ldots, b_2$. The locally optimal b_2' is one that maximizes the mean profit rate less the costs of subcontracting, purchasing and producing components, and inventory, which, in view of the discussion above, is written as

$$J_2(b_2) = \lambda p_2 - \lambda P_1(0)s_1 - \lambda[1 - P_1(0)](p_1 + c_1) - \sum_{n=1}^{b_2} nP_1(n)h_i$$

The function $J_2(b_2)$ is concave [8], and the locally optimal b_2' can be found by trial-and-error.

Next, manufacturer M_1 uses the base stock level b_2' determined by M_2 and computes a locally optimal b_1' by maximizing its own profit rate

$$J_1(b_2) = \mathrm{TH}_2 p_1 - \mathrm{TH}_1 c_1 - \mathrm{STH}_1 s_1 - H_1 h_1$$

where TH_1, TH_2, STH_1, and H_1 are functions of the equilibrium probabilities $P(n_1, n_2)$ and are computed similarly as in the GLOBAL case. The actual net profit rate of strategy LOCAL is computed from Eq. (6), setting $b_1 = b_1'$ and $b_2 = b_2'$.

We have also studied other control policies such as the policy that maintains a constant inventory position in the system and the partly lost sales (PLS) policy, which were introduced in Section 2. The analysis is similar to the base stock case and is omitted for brevity.

3.4 Numerical Results

Consider a system shown in Fig. 4 whose standard parameters are: $\lambda = 5.0$, $\mu_1 = \mu_2 = 6.25$, $p_1 = 70$, $p_2 = 100$, $c_1 = 50$, $c_2 = 10$, $s_1 = 60$, $s_2 = 90$, $h_1 = 3$, $h_2 = 8$. We vary some parameters and compute the optimal base stock levels and the overall profit rates achieved by GLOBAL and LOCAL. The results are shown in Table II.

Table II. Optimal base stocks and performance of GLOBAL and LOCAL for various parameters

Parameters	GLOBAL			LOCAL		
	b_1	b_2	Profit rate	b_1	b_2	Profit rate
standard	3	4	151.168	2	3	142.788
$h_1 = 13$	2	4	137.019	1	2	130.744
$s_1 = 80$	4	4	139.495	3	2	133.840
$c_2 = 19$	3	3	108.670	2	1	94.677
$\mu_1 = \mu_2 = 5$	4	5	142.939	3	3	140.175

From the above table and several other experiments we have performed it appears that a global optimization strategy achieves higher profits than individually optimal strategies in supply chains.

4. Conclusions

In this paper, we have presented basic queueing models to support the task of coordinated decision making among the departments of a manufacturing system, as well as between two firms forming a supply chain. From theoretical and numerical results, it appears that managing inventory levels, sales, and quality tolerances jointly achieves higher profit than independently determined control policies.

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