MODELS FOR THE EVALUATION OF THE EFFECTIVENESS OF PRODUCTION CONTROL STRATEGIES FOR SUPPLY-CHAIN MANAGEMENT

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Abstract: This paper presents models of the following Production Control Strategies (PCS): Kanban Control Strategy (KCS), CONWIP, CONWIP/Pull, Basestock Control Strategy (BSCS) and Extended Kanban Control Strategies (EKCS). These models can be used to compare the effectiveness of the various PCS for managing complex supply-chains. Possible future work to be conducted with the developed models in order to analyse the PCS is presented.

Keywords: Production Control Strategies, hybrid Kanban-CONWIP, CONWIP, Supply-Chain Management, Kanban, Basestock, Extended Kanban Control

1 Introduction

The selection, implementation and management of an appropriate Production Control Strategy (PCS) is an important tool to any organisation aiming to adopt a Lean Manufacturing Philosophy. Two recent papers have explored the possibility of utilising Pull-type PCS to manage inventory and production authorisations in a supply-chain. Takahashi et al. [3] compared the performance of Kanban Control Strategy (KCS) with CONWIP and Synchronised CONWIP in a tiered supply-chain with a single node on the top tier and two nodes on the second tier each of which had two supplier nodes on the bottom tier. Ovalle and Crespo-Marquez [2] compared the performance of CONWIP with MRP for issuing production authorisations in a serial supply-chain. The only source of variation in both models was the demand event. Both models assumed that nodes in a supply-chain would produce the authorised quantity after a known lead-time had elapsed. Furthermore, the model presented in [3] assumes that there are no capacity constraints in place at a node in the supply-chain. The calculation of WIP in [3] appears to exclude inventory in a node at the end of a production period and only considers inventory in the output buffers of nodes.

In this paper we develop models of five PCS, namely KCS, CONWIP, hybrid Kanban-CONWIP, Basestock Control Strategy (BSCS) and Extended Kanban Control Strategy (EKCS). The models are applicable to serial and tiered supply-chains and include capacity constraints and production unreliability for the nodes. Inventory in the system is calculated based on the sum of in-production inventory and finished goods inventory at each node.

2 A Structure Model for Simulation Based Analysis of Supply-Chains

Wang and Xu [4] proposed a Structure Model of Manufacturing Systems that can be used in conjunction with simulation to analyse the effectiveness of various production control strategies in manufacturing systems. Wang and Xu [4] proposed that in any manufacturing system, in terms of the flow of material, there are four types of stages: **i.)** initial raw material input stages, **ii.)** serial stages for part processing, **iii.)** assembly stages, using more than one type of component to produce a product, **iv.)** final stages, for final assembly, packaging and supply to market demand. In order to extend the Structure Model proposed by [4] to supply-chains, we find it necessary to define a fifth stage type, which we term a Distribution stage. This is a stage that outputs parts to several different destinations, such as a warehouse that distributes products to several retail outlets.

The Structure Model of Supply-Chains utilises an adjacent matrix of the network graph of a supply-chain to describe the structure and relationships of the stages in the supply-chain. Define:

$$a_{jk} = \begin{cases} 1 & \text{node } j \text{ is previous to node } k \\ 0 & otherwise \end{cases} \tag{1}$$

Then $A = [a_{jk}]_{m\times m}$ is the adjacent matrix of a network with m nodes. For a network of m nodes define B to be the set of initial nodes, then:

$$B = \left\{ k : \sum_{j=1}^{m} a_{jk} = 0 \qquad k = 1, 2, \ldots, m \right\} \tag{2}$$

This equation means that the initial stages are the nodes with a column sum equal to zero. Define C to be the set of serial stages, then:

$$C = \left\{ j : \sum_{k=1}^{m} a_{jk} = 1 \quad \text{and} \quad \sum_{k=1}^{m} a_{kj} = 1 \qquad j = 1, 2, \ldots, m \right\} \tag{3}$$

This equation implies that serial stages can be identified in the adjacent matrix as the nodes with column and row sums both equal to 1. Define D to be the set of assembly stages, then:

$$D = \left\{ k : \sum_{j=1}^{m} a_{jk} > 1 \qquad k = 1, 2, \ldots, m \right\} \tag{4}$$

This equation implies that assembly stages can be identified in the adjacent matrix as the nodes with column sum greater than 1. Define E to be the set of final stages, then:

$$E = \left\{ j : \sum_{k=1}^{m} a_{jk} = 0 \qquad j = 1, 2, \ldots, m \right\} \tag{5}$$

This equation implies that final stages are nodes with row sum equal to zero. Define F to be the set of distribution stages such that:

$$F = \left\{ j : \sum_{k=1}^{m} a_{jk} > 1 \qquad j = 1, 2, \ldots, n \right\} \tag{6}$$

This equation implies that distribution stages are nodes with row sum greater than one. Define S_j to be the set of stages that supply material to stage j:

$$S_j = \{k : a_{kj} = 1 \qquad j = 1, 2, \ldots, m\} \tag{7}$$

If parts processed in stage j can flow to stage k then stage k is a reachable stage of stage j. Define Z to be the reachable matrix of the network:

$$Z = \sum_{l=1}^{\infty} A^l = (I + A)^{(m-1)} \tag{8}$$

where I is the identity matrix.

Let R_j be the set of reachable stages of stage j, then:

$$R_j = \{k : z_{jk} = 1 \qquad k = 1, 2, ..., m\} \qquad j = 1, 2, ..., m \tag{9}$$

A matrix can be utilised to describe the production ratios of parts at each stage in the supply-chain. Let w_{jk} represent the amount of parts that must be produced at node j in order to produce one part at node k. Then $W = [w_{jk}]_{m \times m}$ is the matrix of production ratios for the supply-chain.

2.1 Uncertainty in Demand and Production

Let the minimum and maximum of demand be $D^{\min}$ and $D^{\max}$, the demand in period n be $d(n)$, and the probability of demand be $Pr[d(n) = d], d = D^{\min}, \ldots, D^{\max}$. Define the probability mass function (PMF) of production at stage j in period n as $Pr[P_j(n) = q_j], q_j = 0, \ldots, P_j^{\max}]$. It is assumed that the PMFs for demand and production are time independent and that the following constraints are true:

$$Pr\left[d(n) = D^{\min}\right] + Pr\left[d(n) = D^{\min} + 1\right] + \cdots + Pr\left[d(n) = D^{\max}\right] = 1 \tag{10}$$

$$Pr\left[P_j(n) = 0\right] + Pr\left[P_j(n) = 1\right] + \cdots + Pr\left[P_j(n) = P_j^{\max}\right] = 1 \tag{11}$$

The probability that stage j produces q_j units in period n given that the production authorisation is $PA_j(n)$, i.e. $\Pr\left[P_j(n) = q_j | PA_j(n)\right]$, is given by:

$$\Pr\left[P_j(n) = q_j | PA_j(n)\right] = \Pr\left[P_j(n) = q_j\right], \qquad q_j = 0, 1, \ldots, PA_j(n) - 1 \tag{12}$$

$$\begin{aligned} \Pr\left[P_j(n) = PA_j(n) | PA_j(n)\right] = {} & \Pr\left[P_j(n) = PA_j(n)\right] \\ & + \Pr\left[P_j(n) = PA_j(n) + 1\right] \quad q_j \geq PA_j(n) \\ & + \ldots + \Pr\left[P_j(n) = P_j^{\max}\right] \end{aligned} \tag{13}$$

2.2 State Variables

The output of the supply-chain in period n will be given by:

$$O\left(n\right) = \min\left[I_m\left(n-1\right) + P_m\left(n - L_m\right), d(n) + BL\left(n-1\right)\right] \tag{14}$$

The size of the backlog of unsatisfied orders in period n can be determined by:

$$BL\left(n\right) = \max[0, \left(d\left(n\right) + BL\left(n-1\right)\right) - \left(I_m\left(n-1\right) + P_m\left(n - L_i\right)\right)] \tag{15}$$

The Service Level, or proportion of demand satisfied in period n is given by:

$$SL(n) = \begin{cases} 1 & BL(n) = 0 \\ \frac{d(n)-BL(n)}{d(n)} & BL(n) < d(n) \\ 0 & BL(n) \geq d(n) \end{cases} \tag{16}$$

In period n, inventory in the supply-chain can be in one of two states. Inventory can be held in the output buffers of the nodes or inventory can be in-production at a node. The state of the system in period n will therefore be described by the following equations:

$$I_j(n) = I_j(n-1) - T_j(n) + P_j(n-L_j) \qquad j = 1, 2, \ldots, m-1 \tag{17}$$

$$I_m(n) = \max[0, I_m(n-1) - (d(n) + BL(n-1)) + P_m(n-L_i)] \tag{18}$$

$$WIP_j(n) = WIP_j(n-1) + P_j(n) - P_j(n-L_j) \qquad j = 1, 2, \ldots, m \tag{19}$$

where $T_j(n)$ is the total production of the immediate successors of node j, and is given by:

$$T_j(n) = \sum_{k=1}^{m} a_{jk} w_{jk} P_k(n) \tag{20}$$

Total inventory held by node j, i.e. in-production plus finished goods, in period n is given by:

$$TI_j(n) = I_j(n) + WIP_j(n) \tag{21}$$

Total inventory in the system for a given component, i.e. number of jobs that have entered through input node j and remain in the system, is determined from:

$$TCI_j(n) = TCI_j(n-1) + P_j(n) - w_{jm}O(n) \qquad \forall \quad j \in B \tag{22}$$

Total inventory in the system in period n in terms of the final product will be given by:

$$TWIP(n) = \frac{\sum_{j=1}^{b} \frac{TCI_j}{w_{jm}}}{b} \tag{23}$$

where b is the number of input nodes i.e. the number of elements in B

2.3 Performance Measures

The performance measures for the supply-chain will be the Average Inventory in the system and the Average Service Level achieved by the system after N periods. Average inventory in the system in terms of the final product will be given by:

$$AWIP(N) = \frac{\sum_{n=1}^{N} (TWIP(n))}{N} \tag{24}$$

Average Service Level achieved by the system will be given by:

$$ASL(N) = \frac{\sum_{n=1}^{N} (SL(n))}{N} \tag{25}$$

3 Modelling the Production Control Strategies

This section presents models for KCS, CONWIP, hybrid Kanban-CONWIP, BSCS and EKCS for authorising production in a supply-chain. While we previously defined a distribution node, we will not be concerned with developing equations to model the production authorisation for a distribution node as this would require specific knowledge of the strategy of the distributor when supply is insufficient to meet the demands of its customer nodes. For instance, in such situations a distributor may attempt to divide the supply equally among its customers or it may seek to prioritise orders from specific customers. Before introducing the models we define $I_k^*(n)$ as the minimum of the inventories held in the output buffers of the immediate predecessors of node j in period n, such that:

$$I_k^*(n-1) = \min\left\{\frac{I_k(n-1)}{w_{kj}}\right\} \qquad \forall \quad k \in S_j \tag{26}$$

3.1 Kanban Control Strategy

In a KCS system, production at stage j is authorised by the presence of Kanban cards and parts. When stage j begins production on a part, a Kanban card is attached to the part and travels downstream with the part. When the succeeding stage begins production on the part the Kanban card is removed and passed back to stage j to be available to authorise production of a new part. For KCS the production authorisations for different nodes in the supply-chain in period n can be obtained from the following formulae:

1. For an initial stage, $j \in B$:
$$PA_j(n) = \min\left[K_j - TI_j(n-1), P_j^{\max}\right] \tag{27}$$

2. For a serial stage, $j \in C$:
$$PA_j(n) = \min\left[K_j - TI_j(n-1), \frac{I_k(n-1)}{w_{kj}}, P_j^{\max}\right] \tag{28}$$
where $I_k(n-1)$ is the inventory of upstream stage k in period $n-1$, k is the sole element of the set S_j.

3. For an assembly stage, $j \in D, \quad j \notin E$:
$$PA_j(n) = \min\left[K_j - TI_j(n-1), I_k^*(n-1), P_j^{\max}\right] \tag{29}$$

4. For a final stage, $j \in E$:
$$PA_j(n) = \min\left[K_j - (TI_j(n-1) - O(n)), I_k^*(n-1), P_j^{\max}\right] \tag{30}$$

3.2 CONWIP Control Strategy

In a CONWIP system, a cap (CC) is placed on the amount of inventory that may be in the supply-chain in period n. Jobs may only enter the system once the total inventory in the system has fallen below this limit. Once a job has entered the supply-chain it is pushed through the system to the customer. For CONWIP systems, the production authorisations for different nodes in the supply-chain in period n can be obtained from the following formulae:

1. For an initial stage, $j \in B$:

$$PA_j(n) = \min\left[w_{jm}(CC + O(n)) - TCI_j(n-1), P_j^{\max}\right] \tag{31}$$

2. For a serial stage, $j \in C$:

$$PA_j(n) = \min\left[\frac{I_k(n-1)}{w_{kj}}, P_j^{\max}\right] \tag{32}$$

   where $I_k(n-1)$ is the inventory of upstream stage k in period $n-1$, k is the sole element of the set S_j.

3. For an assembly stage, $j \in D, \quad j \notin E$:

$$PA_j(n) = \min\left[I_k^*(n-1), P_j^{\max}\right] \tag{33}$$

4. For a final stage, $j \in E$ use equation 32 if $j \notin D$ or equation 33 if $j \in D$

3.3 Hybrid Kanban-CONWIP Control Strategy

In a hybrid Kanban-CONWIP Control Strategy production is authorised at input nodes if the total inventory in the system is less than CC and subject to the availibility of sufficient Kanban cards at the node. Production at all other nodes with the exception of the final node is subject to the availability of sufficient inventory in the output buffers of its immediate predecessors and the availability of sufficient Kanban cards at the node. For the final stage production is subject only to the availability of sufficient inventory in the output buffers of its immediate predecessors. Production authorisations for nodes are, therefore, determined by combining the equations used to model $PA_j(n)$ for KCS and CONWIP as follows:

1. For an initial stage, $j \in B$:

$$PA_j(n) = \min\left[K_j - TI_j(n-1), w_{jm}(CC + O(n)) - TCI_j(n-1), P_j^{\max}\right] \tag{34}$$

2. For a serial stage, $j \in C$ use equation 28 from the KCS model

3. For an assembly stage, $j \in D, \quad j \notin E$ use equation 29 from the KCS model

4. For a final stage, $j \in E$, use equation 32 from the CONWIP model if $j \notin D$ or equation 33 from the CONWIP model if $j \in D$

3.4 Basestock Control Strategy and Extended Kanban Control Strategy

In a system employing BSCS, production at stage j in period n is authorised by the presence of demand cards at the node. When a demand occurs the equivalent number of demand cards are dispatched to each node to authorise the production of new parts. When the node begins production of a new part the demand card is destroyed. The production in period n of node j in an EKCS system is constrained by the availability of Kanban and Demand cards. When a demand occurs, as with BSCS, the equivalent number of demand cards are dispatched to each node to authorise the production of new parts. However, before production can be authorised by the presence of a demand card, the demand card must be matched with a Kanban card and an available part. A demand card is destroyed when node j begins production on the part while the associated Kanban card is attached to the part and travels downstream with the part. When the succeeding node begins production on the part the Kanban card is removed and passed back to node j to be available to authorise production of a new part. The number of demand cards available to node j in period n in either a BSCS or EKCS system is given by:

$$DC_j(n) = DC_j(n-1) - P_j(n-1) + w_{jm} d(n) \tag{35}$$

In BSCS systems the production authorisations for different nodes in the supply-chain in period n can be obtained from the following formulae:

1. For an initial stage, $j \in B$:

$$PA_j(n) = \min\left[DC_j(n), P_j^{\max}\right] \tag{36}$$

2. For a serial stage, $j \in C$:

$$PA_j(n) = \min\left[DC_j(n), \frac{I_k(n-1)}{w_{kj}}, P_j^{\max}\right] \tag{37}$$

3. For an assembly stage, $j \in D, \quad j \notin E$,

$$PA_j(n) = \min\left[DC_j(n), I_k^*(n-1), P_j^{\max}\right] \tag{38}$$

4. For a final stage $j \in E$: use equation 37 if $j \notin D$ or equation 38 if $j \in D$.

In EKCS systems the production authorisations for different nodes in the supply-chain in period n can be obtained from the following formulae:

1. For an initial stage, $j \in B$:

$$PA_j(n) = \min\left[K_j - TI_j(n-1), DC_j(n), P_j^{\max}\right] \tag{39}$$

2. For a serial stage, $j \in C$:

$$PA_j(n) = \min\left[K_j - TI_j(n-1), DC_j(n), \frac{I_k(n-1)}{w_{kj}}, P_j^{\max}\right] \tag{40}$$

3. For an assembly stage, $j \in D, \quad j \notin E$,

$$PA_j(n) = \min\left[K_j - TI_j(n-1), DC_j(n), I_k^*(n-1), P_j^{\max}\right] \tag{41}$$

4. For a final stage, $j \in E$: use equation 40 if $j \notin D$ or equation 41 if $j \in D$.

4 Conclusions

In this paper we have extended the Structure Model for Manufacturing Systems developed in [4] so that it can be used to describe complex supply-chain networks. We have presented mathematical models for several PCS based on the Structure Model for Supply-Chains. We have also presented equations for production and demand unreliability, state variables and performance measures. The mathematical models that we have presented differ from the models presented in [3, 2] in that **i.)** they are applicable to both serial and tiered, **ii.)** production unreliability at the nodes has been modelled **iii.)** nodes are subject to capacity constraints, **iv.)** total inventory in the supply-chain includes in-production inventory at the nodes and **v.)** in addition to modelling KCS and CONWIP, we have presented models for hybrid Kanban-CONWIP, BSCS and EKCS.

These mathematical models can be used to model the behaviour and decision making of the PCS in complex supply-chains and monitor their performances in terms of average WIP and Service Levels achieved. This will allow the performances of the PCS to be compared more comprehensively that has been undertaken in the literature to date. Takahashi et al. [3] compared the performance of CONWIP with KCS and Ovalle and Crespo-Marquez [2] compared the performance of CONWIP with MRP. We aim to translate these mathematical models into discrete event simulation models and utilise a Multi-objective Pareto-Optimal Genetic Algorithm developed by Kernan and Geraghty [1] to conduct experiments to determine the comparative performances of the PCS for managing complex supply-chains.

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