PERFORMANCE EVALUATION OF PRODUCTION LINES WITH FINITE BUFFER CAPACITY PRODUCING TWO DIFFERENT PRODUCTS [1]

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Abstract: The paper presents an approximate analytical method for the performance evaluation of a production line with finite buffer capacity, multiple failure modes and multiple part types. The paper present a solution to a class of problems where flexible machines take different parts to process from distinct dedicated input buffers and deposit produced parts into distinct dedicated output buffers with finite capacity. This paper considers the case of two part types processed in the line, but the method can be extended to the case of n part types. Also the solution is developed for deterministic processing times of the machines which are all identical and are assumed to be scaled to unity. The approach however is amenable of extension to the case of inhomogeneous deterministic processing times. The proposed method is based on the approximate evaluation of the performance of the k-machine line by the evaluation of 2(k-1) two-machine lines. An algorithm inspired to the DDX algorithm has been developed and some preliminary numerical experiments are reported.

Keywords: flow lines, performance evaluation, multiple part types.

1 Introduction

Given the increasing flexibility of manufacturing machines and assembly station it is rather frequent that more than one part type is produced in a single production line. Also, in automated systems machines are normally connected by accumulating conveyors which act as finite capacity buffers. Existing analytical techniques do not allow to model such systems; indeed classical analytical technique allow to model multiclass systems but do not consider finite capacity buffers while approximate analytical techniques developed to model transfer lines do not take into account different part types. The paper presents a solution to a class of problems of this type where flexible machines take different parts to process from distinct dedicated input buffers and deposit produced parts into distinct dedicated output buffers with finite capacity. By dedicated input and output buffers we intend buffers that can store only one part type. The proposed solution is developed for the case of two part types, however the approach is amenable of extension to the multiple part type case. Also the solution is developed for deterministic processing times of the machines which are all identical and are assumed to be scaled to unity. The approach however is amenable of extension to the case of inhomogeneous deterministic processing times.

A typical system of the proposed class is represented in Figure 1 In this case machine M1, M2, M3, M6, M7 are dedicated machines i.e. they can produce only one part type. On the contrary machines M4 and M5 are flexible machines and can produce both part types. The selection of which type of part to produce depends on the state of the system and on a dispatching rule. If the upstream buffer of one part type is empty or the downstream buffer is full, the machine will

[1] The authors would like to thank S. B. Gershwin for the ideas he provided on this topic.

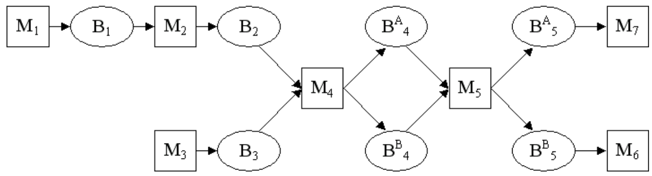

Figure 1: Example of a system producing two part types

produce the other part type. If both the part types are either blocked or starved, the machine will not produce. If both the parts can be produced, than the machine will produce part type A with probability α_i^A and part type B with probability α_i^B.
It is important to notice that the proposed system is quite different from assembly/disassembly systems [2], [1]. Indeed in assembly/disassembly systems assembly machines take contemporarily different parts from different input buffers to produce a single subassembly while disassembly machines from one subassembly produce contemporarily different components that are put into different buffers. In either case there is no selection of components to work on but they are all contemporarily involved in the process. On the contrary, in the described system a flexible machine selects a single component to work on.
The proposed system is also different from fork and join networks [3], [4]. Indeed in fork and join networks each machine can either take the input from different buffers to produce an undifferentiated product or take in input an undifferentiated product and place it after processing in different buffers. On the contrary in the described system flexible machines take in input different parts from different buffers and produce different products placed in the corresponding buffers. In other words the identity of the part is not lost within the machine.
The problem presented in this paper has been originally stated by S.B. Gershwin and addressed by Nemec [5]. The original statement however considers a priority rule between the parts and therefore when both part types can be produced, the part type with the highest priority is selected. This would correspond in our statement to the case of $\alpha^A = 1, \alpha^B = 0$. The solution approach adopted in Nemec is heavily dependent on the original problem statement because parts are treated differently depending on the priority. In the proposed approach on the contrary all the parts are considered in the same way.
It is interesting to consider the fact that the described problem, which has been inspired by automated production system, is similar to other relevant problems that can be addressed with the same methodology. In particular it is interesting to consider the case of production networks where different enterprizes cooperate to produce complex products. In this case each enterprize of the network can be modelled as a flexible machine while input and output storages can be modelled as buffers.

2 Outline of the method

In this paper we consider transfer lines composed of K machines in which two distinct part types (type A and type B) are processed in certain ratios. Both part types follow a linear path through the system since they are processed by all the M_i machines (with $i = 1, ..., K$), starting from the first one and finishing to the last machine after which they leave the system. Adjacent machines are separated by two different buffers B_i^A and B_i^B with limited capacities dedicated to temporally store parts of types A and B respectively. Buffer capacities between machines M_i and M_{i+1} are denoted with N_i^A and N_i^B for part types A and B respectively. Machine M_i of the system works part type A and part type B in the ratios α_i^A and α_i^B when is not blocked or starved.
Machines are multiple failure mode machines, i.e. they are unreliable and can fail in F_i different

modes as assumed in [6]; we denote with $p_{i,j}$ the probability of failure of machine M_i in mode j and with $r_{i,j}$ the probability of repair of machine M_i failed in mode j (with $j = 1, ..., F_i$).

A detailed list of the assumptions used in the proposed model is described in the following; assumptions regard the behavior of the machines and describe in particular how failures can occur and how machines select the part type to produce on the basis of blocking and starvation that characterize the part flow in the system.

- The first machine is never starved, i.e. there is an infinite number of pieces of both part types waiting for being processed in the system.
- The last machine is never blocked, i.e. there is an infinite space downstream the system where it is always possible to store pieces processed by the system.
- Blocking before service (BBS) is assumed for the machines.
- If buffer B_i^A (B_i^B) is full then machine M_i will process part type B (A) if possible.
- If buffer B_{i-1}^A (B_{i-1}^B) is empty then machine M_i will process part type B (A) if possible.
- If for a given machine both the upstream buffers are not empty and both the downstream buffers are not full the machine will produce a part of type A with probability α_i^A and a part of type B with probability α_i^B ($\alpha_i^A + \alpha_i^B = 1$).
- Operation dependent failures are assumed, that is machines can only fail if they are not down, not blocked and not starved.
- A given machine M_i can fail in F_i different failure modes.
- At a given time a machine can be failed in only one mode and cannot enter in a different failure mode.
- Mean time between failures (MTTF) and mean time to repair (MTTR) of machine failures are geometrically distributed and their average values are equal to $1/p_{i,j}$ and $1/r_{i,j}$ respectively (with $i = 1, ..., K$ and $j = 1, ..., F_i$).

The method evaluates the performance measures of the systems described in the previous section by using a generalization of the decomposition technique proposed in [6]. The method can also be used in principle with the decomposition technique proposed in [7]. The analyzed system is decomposed into $K - 1$ sets of two-machine lines that together represent the behavior of the system. Each set is composed of two different two-machine lines or building blocks, each one modelling the flow of one of the two part types in the system (Figure 2). In other words the method creates a two-machine line for each buffer of the original line; each building block is composed of two pseudo machines and one intermediary buffer. The upstream machine represents the behavior of the portion of the system that precedes, in the original line, the corresponding buffer considered in the building block. In the same way the downstream machine represents the behavior of the portion of the system that follows, in the original line, the corresponding buffer considered in the building block. The idea is to analyze simple building blocks, easy to study with existing techniques, instead of the complex original system. In such a way the complexity of the analysis is reduced to study several two-machine lines instead of a long production line. However, the different two-machine lines are not independent and have to be analyzed by means of decomposition equations. To do this, the parameters of the pseudo machines are calculated so that the flow of parts in the buffers of the decomposed systems closely matches the flow in the corresponding buffers of the original line.

Therefore, for buffers B_i^A and B_i^B of the original line, two building blocks (Figure 2) are created. The first building block models the flow of type A parts and is composed of the pseudo

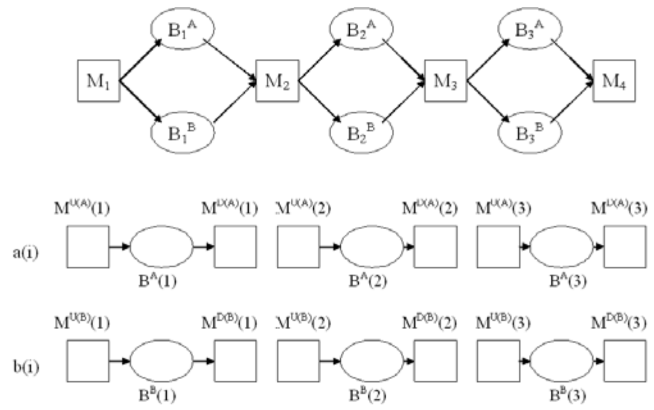

Figure 2: Decomposition of the original line

upstream machine $M^{U(A)}(i)$, the pseudo downstream machine $M^{D(A)}(i)$ and the buffer $B^A(i)$. These two pseudo-machines together with the buffer form the building block $a(i)$. The second building block models the flow of type B parts and is composed of the pseudo upstream machine $M^{U(B)}(i)$, the pseudo downstream machine $M^{D(B)}(i)$ and the buffer $B^B(i)$. These two pseudo-machines together with the buffer form the building block $b(i)$. To model the interruptions of flow through the buffers of the original line, failure rates of different modes are associated to each pseudo-machine. In the following we will consider the case of the upstream pseudo machines. A similar reasoning applies to the downstream pseudo machines.

Interruptions of flow due to a failure in the machine $M(i)$ of the original line are modelled assigning to the upstream pseudo-machines local failure modes with probability of failure p_{i,f_i} and probability of repair r_{i,f_i} (i.e. the same as the ones in the original line).

To mimic the interruptions of flow due to starvation, remote failure modes are introduced and assigned to the upstream pseudo-machine of the building block, namely $M^{U(A)}(i)$ and $M^{U(B)}(i)$. These remote failures have probabilities $p_{j,f_1}^{V(A)}$ and $p_{k,f_2}^{V(B)}$ and probabilities of repair $r_{j,f_1}^{V(A)}$ and $r_{k,f_2}^{V(B)}$ where $j, = 1...i-1$, $k = 1...i-1$ indicate the machines of the original line that actually failed (and are therefore responsible for the starvation) and $f_1 = 1...F_j$, $f_2 = 1...F_k$ indicate the failure modes in which that machines failed. For these remote failure modes, we assume that the repair probabilities are identical to the repair probabilities of the machine of the original line that actually failed. On the other hand, the probability of failure for these remote modes are not known and must be evaluated by using decomposition equation.

The described failure modes follow exactly the approach described in [6] to predict the performance of a transfer line producing only one part type.

To model the interactions between the parts competing for the same machines, in addition to the described failure modes, a new failure mode has been introduced and assigned to each pseudo-machine of the building blocks. This new failure mode has been called *competition failure* and mimics the situation in which a machine does not produce a given part type because it is busy producing the other part type. This new failure mode has probability of failure $p_{j,F_j+1}^{V(A)}$, $p_{k,F_k+1}^{V(B)}$ and probability of repair $r_{j,F_j+1}^{V(A)}$, $r_{k,F_k+1}^{V(B)}$ for part types A and B respectively. Another issue generated by the presence of two part types is that even if a machine is starved or blocked under the point of view of a given part type, it can produce the other part type and can fail while producing that part type. Therefore, the probabilities of local failures must be adjusted to take into account this situation.

In order to estimate failure and repair rate of the competition failure and to adjust local failure probabilities, it is necessary to introduce a model of a *combined pseudo machine* $M^U(i)$ producing two part types.

The solution approach is based on the analysis of all the states in which the combined pseudo machine $M^U(i)$ can be and on the solution of the Markov chain of the combined pseudo machine. In this Markov chain some transition probabilities are not known, however, the probabilities of some states of this Markov chain can be obtained from the results of the upstream building blocks. Indeed, these values are obtained by means of decomposition equations which are a generalization of the ones derived in [6]. Therefore, at the end, it is possible to solve a linear system of equations which allow to evaluate both the unknown transition probabilities and the probabilities of all the states of the Markov chain.

The probabilities obtained for the various states of the combined pseudo machine are then used to build two separate models, one for each upstream pseudo-machine of the two building blocks ($M^{U(A)}(i)$, $M^{U(B)}(i)$). By studying these two models it is then possible to calculate new local failure parameters for the pseudo-machines, considering the possibility for each machine of going down due to a failure occurred processing the other part. In addition, it is possible to find the probabilities of failure and repair of the competition failure. These parameter completely define the pseudo machines and allow in turn to evaluate the building blocks.

3 Detailed description of the method

3.1 Combined pseudo machine model

The picture below (Figure 3), represents the Markov chain of the combined pseudo machine $M^U(i)$. To simplify the picture, all the states of the same type are grouped into a unique state without considering different failure modes. Obviously in writing the equations it is important to distinguish all the different failure modes, to correctly evaluate state probabilities. Each state in the combined pseudo machine is defined by two state variables, one for the pseudo-machine of the line A and the other for the pseudo-machine of the line B. Each state variable can assume four values that, if we consider an upstream machine, are: working (W), down in local mode (R), down in remote mode (V) and blocked (B) . In total there are 16 possible states. It must be remembered that a combined pseudo machine is related to only one physical machine (the local machine) of the original line. As a consequence, a combined pseudo machine cannot be both working a part type while being down in local mode for the other part type therefore the two states W^AR^B, R^AW^B are not feasible and are not represented in the picture. Also the state R^AR^B represents a situation where the local machine is down and therefore cannot produce either A or B. We call this state pure local down state and we rename it R. Finally the state W^AW^B represents a state were for both part types no failure, local or remote, is present therefore the machine can produce either A or B.

In the following, some key characteristics of the Markov chain of the combined pseudo machine $M^U(i)$ (Figure 3) are discussed:

- If the combined pseudo machine is in state W^AV^B and while producing part type A (part type B cannot be produced because it is down in remote mode) it fails, it goes in state R^AV^B. This means that the combined pseudo machine is down both in local mode and in remote mode. From this state it can go either to a pure local down state R if the remote failure is repaired or back to W^AV^B if the local failure is repaired or to W^AW^B if both local and remote failures are repaired. A similar reasoning applies to the states W^AB^B, V^AW^B, B^AW^B.

- If the combined pseudo machine is in pure local down state, by repairing the local failure it always enters the W^AW^B state.

- When the combined pseudo machine is in state W^AW^B it can process A or B depending on the processing rate α_i^A and α_i^B ($\alpha_i^A + \alpha_i^B = 1$). Therefore from state W^AW^B,

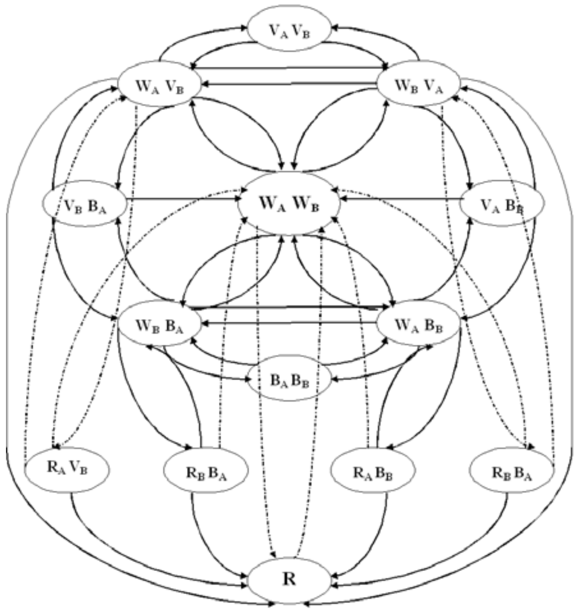

Figure 3: Markov chain of the combined pseudo machine

since only one of the two part types is produced, it is not possible to go to states $B^AB^B, B^AS^B, S^AB^B, S^AS^B$ (because if a part type is not produced it is not possible to have blocking or starvation for that part type).

- During a time interval a given machine of the line can at most process one part; therefore it is impossible to move from states S^AS^B or B^AB^B to state W^AW^B. Indeed these transitions would imply that either one machine upstream in case of starvation or one machine downstream in case of blocking processes two parts during the same time interval.

As already mentioned, in Figure 3, to simplify the picture, all the states of the same type are grouped into a unique state without considering different failure modes. The probability of these 14 grouped states is therefore the sum of the probabilities of the disaggregated states considering all the failure modes. It must be noticed that for each machine of the line a competition failure is added to the real failures of the machine to take into account the presence of two part types. Therefore machine M_i has $F_i + 1$ failure modes.

It must also be noticed that in this Markov chain not all the transition probabilities are known. Indeed the values of $p_{j,f_1}^{V(A)}$ and $p_{k,f_2}^{V(B)}$ cannot be derived directly from the original line and therefore they must be found using appropriate equations. In the following the 14 sets of equations required to evaluate the probabilities of the various states plus the equations required to evaluate the unknown transition probabilities are provided for the upstream combined pseudo machine.

Since in all the lines the flow of material has to be the same, we can write the conservation of flow equations, one for line A and one for line B:

$$W^AW^B\alpha_A + \sum_{j=1}^{i-1}\sum_{f=1}^{F_j+1} W^AV_{j,f}^B + \sum_{k=i+1}^{K}\sum_{f=1}^{F_k+1} W^AB_{k,f}^B = E^A(i-1) \tag{1}$$

$$W^AW^B\alpha_B + \sum_{j=1}^{i-1}\sum_{f=1}^{F_j+1} W^BV_{j,f}^A + \sum_{k=i+1}^{K}\sum_{f=1}^{F_j+1} W^BB_{k,f}^A = E^B(i-1) \tag{2}$$

As originally proposed in [6], we introduce remote down states for the combined pseudo machine to mimic starvation. Therefore the probability of being down in remote mode of the combined pseudo machine has to be equal to the probability of starvation of the preceding building block.

$$\sum_{k=1}^{i-1}\sum_{f_2=1}^{F_k+1} V^A_{j,f_1}V^B_{k,f_2} + \sum_{k=i+1}^{K}\sum_{f_2=1}^{F_k+1} V^A_{j,f_1}B^B_{k,f_2} + W^B V^A_{j,f_1} + \sum_{g=1}^{F_i} R^B_{i,g}V^A_{j,f_1} = Ps^A_{j,f_1}(i-1) \qquad (3)$$
$$j = 1...i-1, f_1 = 1...F_j+1$$

$$\sum_{j=1}^{i-1}\sum_{f_1=1}^{F_j+1} V^A_{j,f_1}V^B_{k,f_2} + \sum_{j=i+1}^{K}\sum_{f_1=1}^{F_j+1} B^A_{j,f_1}V^B_{k,f_2} + W^A V^B_{k,f_2} + \sum_{g=1}^{F_i} R^A_{i,g}V^B_{k,f_2} = Ps^B_{k,f_2}(i-1) \qquad (4)$$
$$k = 1...i-1, f_2 = 1...F_k+1$$

Given the fact that the upstream combined pseudo machine of building block i must be coherent with the building blocks $a(i)$ and $b(i)$ introduced for part type A and B respectively, we can write the following equations related to the probability that the combined pseudo machine is blocked:

$$\sum_{k=1}^{i-1}\sum_{f_2=1}^{F_k+1} V^B_{k,f_2}B^A_{j,f_1} + \sum_{k=i+1}^{K}\sum_{f_2=1}^{F_k+1} B^A_{j,f_1}B^B_{k,f_2} + \sum_{g=1}^{F_i} R^B_{i,g}B^A_{j,f_1} + W^B B^A_{j,f_1} = Pb^A_{j,f_1}(i) \qquad (5)$$
$$j = i+1...K; f_1 = 1...F_j+1$$

$$\sum_{j=1}^{i-1}\sum_{f_1=1}^{F_j+1} S^A_{j,f_1}B^B_{k,f_2} + \sum_{j=i+1}^{K}\sum_{f_1=1}^{F_j+1} B^A_{j,f_1}B^B_{k,f_2} + \sum_{g=1}^{F_i} R^A_{i,g}B^B_{k,f_2} + W^A B^B_{k,f_2} = Pb^B_{k,f_2}(i) \qquad (6)$$
$$k = i+1...K; f_2 = 1...F_k$$

Considering the states R^AV^B , R^AB^B , R^BV^A , R^BB^A , we can write node equations balancing the probability of entering these states with the probability of exiting the same states.

$$W^A V^B_{j,f}p_{i,g}(1 - r^{V(B)}_{j,f}) = R^A_{i,g}V^B_{j,f}r^{V(B)}_{j,f} + R^A_{i,g}V^B_{j,f}r_{i,g}(1 - r^{V(B)}_{j,f}) \qquad (7)$$
$$g = 1...F_i, j = 1...i-1, f = 1...F_j+1$$

$$W^B V^A_{j,f}p_{i,g}(1 - r^{V(A)}_{j,f}) = R^B_{i,g}V^A_{j,f}r^{V(A)}_{j,f} + R^B_{i,g}V^A_{j,f}r_{i,g}(1 - r^{V(A)}_{j,f}) \qquad (8)$$
$$g = 1...F_i, j = 1...i-1, f = 1...F_j+1$$

$$W^A B^B_{j,f}p_{i,g}(1 - r^{B(B)}_{j,f}) = R^A_{i,g}B^B_{j,f}r^{B(B)}_{j,f} + R^A_{i,g}B^B_{j,f}r_{i,g}(1 - r^{B(B)}_{j,f}) \qquad (9)$$
$$g = 1...F_i, j = i+1...K, f = 1...F_j+1$$

$$W^B B^A_{j,f}p_{i,g}(1 - r^{B(A)}_{j,f}) = R^B_{i,g}B^A_{j,f}r^{B(A)}_{j,f} + R^B_{i,g}B^A_{j,f}r_{i,g}(1 - r^{B(A)}_{j,f}) \qquad (10)$$
$$g = 1...F_i; j = i+1...K; f = 1...F_j+1$$

Considering the states V^AV^B , V^AB^B , B^AV^B , B^AB^B we can write node equations balancing the probability of entering these states with the probability of leaving the same states

$$W^A V^B_{k,f_2}p^{V(A)}_{j,f_1}(1 - r^{V(B)}_{k,f_2}) + W^B V^A_{j,f_1}p^{V(B)}_{k,f_2}(1 - r^{V(A)}_{j,f_1}) = V^A_{j,f_1}V^B_{k,f_2}(r^{V(A)}_{j,f_1} + r^{V(B)}_{k,f_2}) \qquad (11)$$
$$j = 1...i-1; f_1 = 1...F_j+1; k = 1...i-1; f_2 = 1...F_k+1$$

$$W^A B^B_{k,f_2}p^{B(A)}_{j,f_1}(1 - r^{B(B)}_{k,f_2}) + W^B B^A_{j,f_1}p^{B(B)}_{k,f_2}(1 - r^{B(A)}_{j,f_1}) = B^A_{j,f_1}B^B_{k,f_2}(r^{B(A)}_{j,f_1} + r^{B(B)}_{k,f_2}) \qquad (12)$$
$$j = i+1...K; f_1 = 1...F_j+1; k = i+1...K; f_2 = 1...F_k+1$$

$$W^A B^B_{k,f_2}p^{V(A)}_{j,f_1}(1 - r^{B(B)}_{k,f_2}) + W^B V^A_{j,f_1}p^{B(B)}_{k,f_2}(1 - r^{V(A)}_{j,f_1}) = V^A_{j,f_1}B^B_{k,f_2}(r^{V(A)}_{j,f_1} + r^{B(B)}_{k,f_2})$$

$$j=1...i-1; f_1=1...F_j+1; k=i+1...K; f_2=1...F_k+1$$

$$W^A V^B_{k,f_2} p^{B(A)}_{j,f_1}(1-r^{V(B)}_{k,f_2}) + W^B B^A_{j,f_1} p^{V(B)}_{k,f_2}(1-r^{B(A)}_{j,f_1}) = V^B_{k,f_2} B^A_{j,f_1}(r^{B(A)}_{j,f_1}+r^{V(B)}_{k,f_2}) \quad (13)$$

$$j=i+1...K; f_1=1...F_j+1; k=1...i-1; f_2=1...F_k+1$$

Considering the set of states R , $R^A V^B$, $R^A B^B$, $R^B V^A$, $R^B B^A$, we can write equations balancing the probability of entering this set of states with the probability of leaving this set of states.

$$(R_{i,g} + \sum_{j=1}^{i-1}\sum_{f_1=1}^{F_j} R^B_{i,g} V^A_{j,f_1} + \sum_{k=1}^{i-1}\sum_{f_2=1}^{F_k} R^A_{i,g} V^B_{k,f_2} + \sum_{k=i+1}^{K}\sum_{f_2=1}^{F_k+1} R^A_{i,g} B^B_{k,f_2} + \sum_{j=i+1}^{K}\sum_{f_1=1}^{F_j+1} R^B_{i,g} B^A_{j,f_1}) r_{i,g} = \quad (14)$$

$$p_{i,f_i}(W^A W^B + \sum_{j=1}^{i-1}\sum_{f_1=1}^{F_j+1} W^B V^A_{j,f_1} + \sum_{k=1}^{i-1}\sum_{f_2=1}^{F_k+1} W^A V^B_{k,f_2} + \sum_{j=i+1}^{K}\sum_{f_1=1}^{F_j+1} W^B B^A_{j,f_1} + \sum_{k=i+1}^{K}\sum_{f_2=1}^{F_k+1} W^A B^B_{k,f_2})$$

$$g=1...F_i$$

In order to calculate the unknown transition probabilities we first write the node equation for node $W^A V^B$ and then after some manipulation we obtain:

$$p^{V(B)}_{k,f_2} = \frac{Ps^B_{k,f_2}(i-1)}{E^B(i-1)} r^{V(B)}_{k,f_2} \qquad k=1...K; f_2=1...F_k+1 \quad (15)$$

$$p^{V(A)}_{j,f_1} = \frac{Ps^A_{j,f_1}(i-1)}{E^A(i-1))} r^{V(A)}_{j,f_1} \qquad j=1...i-1; f_1=1...F_j+1 \quad (16)$$

3.2 Pseudo machine models

Once obtained the probability of being in each state of the combined pseudo machine, we can build two models, one for each pseudo-machine $M^{U(A)}(i)$ and $M^{U(B)}(i)$. This results in a five state model. The probability of each state is obtained by summing the values calculated with the previous system. For the pseudo-machines $M^{U(A)}(i)$ and $M^{U(B)}(i)$, we have:

$$W^{A^*} = W^A W^B \alpha^A + W^A V^B + W^A B^B$$
$$V^{A^*} = V^A V^B + V^A B^B + W^B V^A + R^B V^A$$
$$R^{A^*} = R + R^A V^B + R^A B^B$$
$$B^{A^*} = W^B B^A + V^B B^A + B^A B^B + R^B B^A$$
$$W^B = W^A W^B \alpha_B$$

$$W^{B^*} = W^A W^B \alpha^B + W^B V^A + W^B B^A$$
$$V^{B^*} = V^A V^B + V^B B^A + W^A V^B + R^A V^B$$
$$R^{B^*} = R + R^B V^A + R^B B^A$$
$$B^{B^*} = W^A B^B + V^A B^B + B^B B^A + R^A B^B$$
$$W^A = W^A W^B \alpha_A$$

Having these two models, one for each pseudo-machine, and knowing all the state probabilities, it is possible to calculate local failure parameters for the pseudo machine. It must be noticed that the probability of entering in local failure is higher than the one of the corresponding machine in the original line because we take into account the probability of failing while producing the other part type. In addition since we know the probability of being in the competition failure state (that models the situation in which the pseudo-machine does not produce because the other pseudo-machine is producing) we can use it to evaluate the parameters of the competition failure. To estimate the parameter of the competition failure we introduce the assumption that the repair rate of the competition failure must be as high as possible to allow a frequent switch in the production of the two part types. In doing this we consider the constraint that the sum of all the failure probabilities for the various failure modes must be lower than 1. We can evaluate these parameters utilizing balancing equation to nodes R^{A^*} and W^B

$$p^{A^*}_{i,g} = \frac{R^{A^*}}{W^{A^*}} r_{i,g} = \frac{R^{A^*}}{E^A(i-1)} r_{i,g} \quad g=1...F_i \quad (17)$$

$$p_{i,F_i+1}^{A*} = \frac{W^B}{W^{A*}} r_{i,F_i+1}^{A*} \quad (18)$$

There are two cases for deciding the value of p_{i,F_i+1}^{A*} and r_{i,F_i+1}^{A*}:

$$W^B < W^{A^*} \quad \Rightarrow r_{i,F_i+1}^{A*} = 1 \quad p_{i,F_i+1}^{A*} = W^B / W^{A^*} \quad (19)$$
$$W^B >= W^{A^*} \quad \Rightarrow p_{i,F_i+1}^{A*} = \alpha_B \quad r_{i,F_i+1}^{A*} = W^{A^*} / W^B p_{i,F_i+1}^{A*} \quad (20)$$

In a similar way we can find for machine $M^{U(B)}(i)$ the values of $p_{i,g}^{B^*}$, $p_{i,F_i+1}^{B^*}$ and $r_{i,F_i+1}^{B^*}$. Once local, remote and competition failure probabilities are evaluated they can be used within the building blocks $a(i)$ and $b(i)$.

4 Preliminary numerical results

In order to evaluate the precision of the proposed method, simple cases are considered in the preliminary numerical analysis reported in this Section. In particular a two-machine/two-buffer system and a three-machine/four-buffers are studied with the objective of estimating their average throughput and average buffer levels. Also the number of iterations necessary to obtain performance parameters are investigated to test the convergence of the method. Parameters of the first system analyzed are reported in Table 1.

CASE 1	p_i	r_i	$B^A(i)$	$B^B(i)$	α_i^A	α_i^B
$i = 1$	0, 23	0, 4	4	6	0, 6	0, 4
$i = 2$	0, 37	0, 3			0, 6	0, 4

Table 1: Two-machine/two buffer line: system parameters.

	$E(i)$	$\overline{n}_b$	$Ps(i)$	$Pb(i)$
TYPE A	0, 264	3, 223	0, 011	0, 457
TYPE B	0, 183	5, 265	0, 003	0, 537

Table 2: Two-machine/two buffer line: average throughput and buffer levels.

The results reported in Table 2 have been obtained by a first implementation of the proposed method. The number of iterations necessary to arrive to a convergence in this particular case is equal to 9. It is worthwhile to notice how the ratio between throughput of part types A and B changes from the the initial values of α_i^A and α_i^B used in the system and reported in Table 1. Indeed, throughput ratios effectively obtained by the system can be calculated by using numerical results in Table 2 as follows:

$$\begin{aligned} \alpha*^A &= \frac{E^A}{E^A+E^B} = 0,591 \\ \alpha*^B &= \frac{E^B}{E^A+E^B} = 0,409 \end{aligned} \quad (21)$$

We can see that using resulting values of throughput, the values of $\alpha*^A$and $\alpha*^B$ are quite different from the values adopted in the system. This is due to the fact that the occurrence of blocking is different for part type A or B depending on their relative buffer capacities. Indeed, in this case the flow of part B, which has a larger buffer capacity than product A, is characterized by a frequency of blocking that is lower than that of product A (see Table 1). It would be important as a future development of the method to develop a method able to find out the values of precedence parameters for each machine of the line, starting from the values that we want effectively to obtain from the system.

The other case considered in this paper is a line with three machines and four buffers. The parameters of this system are reported in Table 3. Also in this case results have been obtained in few iterations. Effective values of $\alpha *^A$ and $\alpha *^B$ are equal respectively to $0,585$ and $0,415$. The difference between these values and the ones used in the system is greater than the one measured in the previous case due to the fact that the line is longer and therefore occurrence of blocking and starvation increases during the production.

CASE 2	p_i	r_i	$B^A(i)$	$B^B(i)$	α_i^A	α_i^B
$i = 1$	$0,12$	$0,35$	6	8	$0,6$	$0,4$
$i = 2$	$0,16$	$0,3$	6	10	$0,6$	$0,4$
$i = 3$	$0,08$	$0,5$			$0,6$	$0,4$

Table 3: Three-machine/four buffer line: system paremeters.

	$E^A(i)$	$E^B(i)$	$\overline{n}_b^A(i)$	$\overline{n}_b^B(i)$	$Ps^A(i)$	$Pb^A(i)$	$Ps^B(i)$	$Pb^B(i)$
$i = 1$	$0,381$	$0,270$	4.500	$6,861$	$0,018$	$0,293$	$0,001$	$0,362$
$i = 2$	$0,381$	$0,270$	$0,737$	$0,565$	$0,467$	$0,001$	$0,556$	$1.6E-06$

Table 4: Three-machine/four buffer line: average throughput and buffer levels.

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