BASE STOCK POLICIES WITH SOME UNRELIABLE ADVANCE DEMAND INFORMATION

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Abstract: We investigate the impact of variability and uncertainty in the amount of advance demand information (ADI) on the performance of single-stage make-to-stock supply systems. First, we study a basic model of a system in which ADI is neither variable nor uncertain. Specifically, we assume that all customers provide constant reliable ADI in the form of firm orders, which are placed a fixed amount of time in advance of their respective due dates. Then, we extend the basic model by assuming that the system has some unreliable ADI. Specifically, we assume that some customers provide unreliable ADI in the form of cancelable reservations, while others provide no ADI at all. Assuming that the reorder policy is a base stock policy with ADI, we investigate the impact of certain system parameters, which are related to the ADI mechanism, on the optimal design parameters and performance of the reorder policy, for the cases where the supply process is modeled as an $M/M/1$, $M/D/1$, $M/M/\infty$ and $M/D/\infty$ queuing system, respectively.

Keywords: base stock; advance demand information; reservation; cancellation.

1 Introduction and Literature Review

There is a general belief among POM researchers and practitioners that obtaining and distributing demand information to all the partners of a supply chain is essential for improving the coordination and ultimately the performance of the supply chain. The benefits of sharing demand information are further amplified when this information is obtained in advance. One way of obtaining *advance demand information* (ADI) is by inciting customers to place their orders ahead of time. This can be accomplished by offering price discounts or service priority to customers who order in advance. In practical situations, however, not all customers who are given the opportunity to order in advance will do so and of those who will order in advance, some may subsequently change or cancel their orders. The aim of this paper is to investigate the impact of variability and uncertainty in the amount of ADI on the performance of single-stage make-to-stock supply systems. In Section 2, we study a basic model of a single-stage make-to-stock supply system in which there is a single class of customers and each customer places a firm order a fixed amount of time before his/her requested due date. We focus our attention to the cases where the supply process is modeled as an $M/M/1$, $M/D/1$, $M/M/\infty$ and $M/D/\infty$ queuing system, respectively. The basic model represents the situation where every customer provides constant and reliable ADI. In Section 3, we extend the basic model by assuming that there are two classes of customers. Each customer in the first class requires immediate service, whereas each customer in the second class makes a reservation for a finished product a fixed amount of time before his requested due date and must confirm (or cancel) this reservation a fixed amount of time prior to this due date. The extended model represents the situation where some customers provide unreliable ADI. The reservation-confirmation mechanism described above can also be viewed as a surrogate for a forecasting system in which there are some confirmed orders in the short term and forecasts of orders in the longer term. In both the basic and the extended model, the reorder policy used is a *base stock policy with ADI* (BSADI) [14], [15]. Finally, in Section 4, we investigate the impact of certain system parameters, which are related to the ADI mechanism, on the optimal design parameters and performance of the system for the four different supply cases considered in Section 2.

Most of the literature on ADI concerns uncapacitated supply systems, i.e. systems where the supply lead times are independent (e.g. [2], [6], [7], [8], [13], [18] and [21]). Work that investigates ADI in capacitated production/inventory systems includes [1], [9], [10], [11] and [20]. Our work is most closely related to [3], [4], [5], [14], [15], [16] and [17]. Specifically, references [3], [4] and [5] present a detailed analysis of a single-stage make-to-stock queue with ADI in the form of firm orders placed a fixed amount of time in advance of their due-dates and investigate how the optimal safety stock varies as a function of this time Reference [14] investigates the structure of the optimal release timing and inventory control decisions for a discrete-time make-to-stock queue with ADI. References [15] and [16] assess the value of ADI for a continuous-time make-to-stock queue with ADI for the cases where customers accept (or do not accept) deliveries earlier than their required due-dates. Reference [17] presents a simulation-based investigation of WIP-controlled BSADI policies for a single-stage and a two-stage make to stock capacitated system. Finally, we should note that make-to-stock supply systems with ADI are structurally similar to assemble-to-order (ATO) systems. Recent results and references on ATO systems can be found in [19].

2 Single-Stage Make-to-Stock Supply System with Constant Reliable ADI

We consider a single-stage make-to-stock supply system with constant reliable ADI, which operates under a BSADI policy as follows. Customer demands arrive for one end-item at a time according to a Poisson process with rate λ, with a constant *demand lead time*, T, in advance of their due dates. Once a customer demand arrives, it cannot be cancelled. The arrival of every customer demand eventually triggers the consumption of an end-item from *finished goods* (FG) inventory and the issuing of an order to replenish FG inventory. There is no setup cost or setup time for issuing a replenishment order and no limit on the number of orders that can be placed per unit time, so a one-for-one replenishment policy is used. The consumption of an end-item from FG inventory is triggered T time units after the arrival of the demand. If no end-items are available at that time, the demand is backordered. The system starts with a *base stock*, S, of end-items in FG inventory. All end-items are uniform and interchangeable; customers do not care which end-item they receive. The time of issuing the replenishment order is determined by offsetting the demand due date by a fixed *planned supply lead time*, L, according to an MRP time-phasing logic. This means that the order is issued with no delay with respect to the corresponding demand arrival time, if $L \geq T$, or with a delay equal to $T - L$ with respect to the demand arrival time, if $L < T$. In other words, the delay in issuing a replenishment order is constant and equal to $\max(0,T - L)$. When the order is issued, a new part is immediately released into the supply system. The model described above is simple but it captures the essentials of the operation of a single-stage make-to-stock supply system with constant reliable ADI, except for lot sizing and multiple product issues, which we purposely keep out of the picture for clarity and simplicity. In what follows, we use the following notation:

I = steady-state FG inventory,
B = steady-state backordered customer demands,
N = steady-state outstanding replenishment orders,
M = steady-state customer demands whose due-dates have not yet expired,
$X = I - B$ = steady-state FG surplus/backlog,
$Z = N - M$ = steady-state outstanding replenishment orders surplus/backlog,
W = steady-state replenishment time, i.e. flow time in the supply process.

It is easy to see that the above quantities satisfy the invariant,

$$Z + X = N - M + I - B = S. \tag{1}$$

We consider a classical optimization problem where the objective is to find the values of S and L that minimize the long run expected average cost of holding and backordering FG inventory for a given demand lead time T,

$$C(T;S,L) = hE[I(T;S,L)] + bE[B(T;S,L)], \tag{2}$$

where h and b are the unit cost rates of holding and backordering FG inventory, respectively. Letting $P_Y(\cdot)$ and $F_Y(\cdot)$ denote the pdf and cdf of a random variable Y, respectively, the long run expected average cost (2) can be expressed in terms of the pdfs of X and Z, respectively, as

$$C(T;S,L) = h\sum_{n=1}^{\infty} nP_X(n) - b\sum_{n=-\infty}^{-1} nP_X(n)\text{, or equivalently,} \tag{3}$$

$$C(T;S,L) = h\sum_{n=0}^{S-1}(S-n)P_Z(n) + b\sum_{n=S+1}^{\infty}(n-S)P_Z(n)\,, \tag{4}$$

where $P_X(n)$ depends on T, S and L and $P_Z(n)$ depends on T and L but not on S. From (3) and (4) it is easy to show that for a fixed L, the optimal base stock $S^*(T;L)$ is the smallest integer S that satisfies

$$F_X(0) = \sum_{n=-\infty}^{0} P_X(n) \le h/(h+b) \text{, or equivalently,} \tag{5}$$

$$F_Z(S) = \sum_{n=-\infty}^{S} P_Z(n) \ge b/(h+b). \tag{6}$$

If $T = 0$, L is irrelevant, since both the consumption of an end-item from FG inventory and the replenishment order are triggered at the demand arrival instant. In this case, $M = 0$, hence $Z = N$. The long run expected average cost (3) or (4) can then be written in terms of the pdf of N as,

$$C(S) = h\sum_{n=0}^{S-1}(S-n)P_N(n) + b\sum_{n=S+1}^{\infty}(n-S)P_N(n), \tag{7}$$

where $P_N(n)$ does not depend on either T, S or L. Similarly, the optimality condition (5) or (6) can be written in terms of $F_N(\cdot)$ as,

$$F_N(S) = \sum_{n=0}^{S} P_N(n) \ge b/(h+b). \tag{8}$$

If $T > 0$, there is a time lag of $\max(0,T-L)$ between the arrival of a demand and the issuing of a replenishment order and a time lag of $T - \max(0,T - L) = \min(T,L)$ between the issuing of a replenishment order and the request for the consumption of an end-item from FG inventory. This implies that any system with $T > L$ behaves exactly like a system with $T = L$, except that the first system has a time lag of $T - L$ with respect to the second system. Moreover, if $T > 0$, $M \neq 0$, hence $Z \neq N$. Following the derivations in [3] – [5], it can be shown that $P_Z(\cdot)$ can be expressed in terms of $P_N(\cdot)$ as

$$P_Z(n) = \begin{cases} \sum_{k=n}^{\infty} P_{T_k^L}(k-n)P_N(k), & n > 0, \\ \sum_{k=0}^{\infty} P_{T_k^L}(k-n)P_N(k), & n \le 0, \end{cases} \tag{9}$$

where $P_{T_k^L}(i)$ is the steady-state conditional probability of having i replenishment completions in a time interval of length $T^L = \min(T,L)$, given that at the beginning of the time interval there are k outstanding replenishment orders in the system. Once $P_Z(\cdot)$ is known, $C(T;S,L)$ and $S^*(T;L)$ can be computed from (4) and (6), respectively; however, obtaining $P_{T_k^L}(\cdot)$ and consequently $P_Z(\cdot)$, $S^*(T;L)$ and the associated cost $C(T;S^*(T;L),L)$ is not trivial. Intuition suggests that for a fixed L, as T increases from zero, $S^*(T;L)$ and $C(T;S^*(T;L),L)$ should decrease until they both drop to their respective minimum levels, $S^*(L;L)$ and $C(L;S^*(L;L),L)$, at $T = L$. For $T > L$, $S^*(T;L)$ and $C(T;S^*(T;L),L)$ should remain constant and equal to $S^*(L;L)$ and $C(L;S^*(L;L),L)$, respectively, because as was mentioned above any system with $T > L$ behaves exactly like a system with $T = L$. The question is what are the optimal parameters L^* and $S^*(L^*;L^*)$?

The only general analytical result related to the above question is Proposition 1 in [15], which states that for any supply system satisfying the following assumption, if the system operates in a make-to-order mode (i.e., with zero base stock) and $T \ge L^*$, the optimal planned supply lead time L^* is the smallest real number L that satisfies

$$F_W(L) \ge b/(h + b). \tag{10}$$

Assumption 1: *All replenishment orders enter the supply system one at the time, remain in the system until they are fulfilled (there is no blocking, balking or reneging), leave one at a time in the order of arrival (FIFO) and do not affect the flow time of previous replenishment orders (lack of anticipation).*

In [15], it is further claimed that under Assumption 1, if the system operates under a make-to-stock policy, L^* is also given by (10). This implies that when the demand lead time switches from zero to some positive value T, such that $T \geq L^*$, where L^* satisfies (10), the optimal supply policy switches from a pure make-to-stock policy with S^* satisfying (8), into a pure make-to-order policy with L^* satisfying (10). Notice the similarity between expressions (8) and (10), which demonstrates the interchangeability of safety stock and safety time.

In what follows we will examine in detail four special cases in which the supply process is modeled as an $M/M/1$, $M/D/1$, $M/M/\infty$ and $M/D/\infty$ queuing system, respectively. The first two cases model a capacitated manufacturing environment where the supply lead times are sequential, and the last two cases model an uncapacitated inventory system where the supply lead times are independent. In all cases, except the case $M/M/\infty$, Assumption 1 is satisfied.

Case 1: *M/M/*1. In case the supply process is modeled as a single-server station with exponential service rate μ, it is well known that $P_N(n) = (1-\rho)\rho^n$, $n = 0, 1, \ldots$, and therefore $F_N(S) = (1-\rho)^{S+1}$, where $\rho = \lambda/\mu$ (e.g. see [12]). The analysis in [3] – [5] and [15] yields the following closed-form expressions for the cost and optimal control policy parameters:

$$C(T;S,L) = h\left(S + \lambda T^L - \frac{\rho}{1-\rho}\right) + (h+b)\frac{\rho^{S+1}}{1-\rho}e^{-\mu T^L(1-\rho)}, \tag{11}$$

$$L^* = \frac{\ln\left[(h+b)/h\right]}{\mu(1-\rho)}, \tag{12}$$

$$S^* = \left\lfloor \hat{S}^*(T) \right\rfloor, \text{ where } \hat{S}^*(T) = \min\left\{0, \frac{\ln\left[h/(h+b)\right]}{\ln\rho} + \frac{\mu(1-\rho)}{\ln\rho}T\right\}. \tag{13}$$

Moreover, for the $M/M/1$ system it is well-known that $F_W(w) = 1 - e^{-(\mu-\lambda)w}$, $w \geq 0$, and $E[W] = 1/\mu(1-\rho)$. Given that the $M/M/1$ system satisfies Assumption 1, the optimal planned supply lead time L^* can be alternatively computed from (10). The result is identical to that given by (12).

Case 2: *M/D/*1. In case the supply process is modeled as a single-server station with deterministic service time $1/\mu$, N has the following distribution [12]:

$$P_N(n) = (1-\rho)\left[\sum_{k=1}^{n} e^{k\rho}(-1)^{n-k}\frac{(k\rho)^{n-k}}{(n-k)!} - \sum_{k=1}^{n-1} e^{k\rho}(-1)^{n-k-1}\frac{(k\rho)^{n-k-1}}{(n-k-1)!}\right],$$

where $\rho = \lambda/\mu$. If $T = 0$, S^* can be computed numerically from (8) after substituting $P_N(n)$ with the above expression. To find $P_{T_k^L}(i)$, $i < k$, $k > 1$, we note that, given that at the beginning of a time interval of length $T^L = \min(T,L)$ there are k outstanding replenishment orders, the number of replenishment completions in this time interval, i, for $i < k$ and $k > 1$, is either $\lfloor \mu T^L \rfloor$, if the percentage of the remaining completion time of the outstanding replenishment order in service at the beginning of the time interval is greater than $\lceil \mu T^L \rceil - \mu T^L$, or $\lceil \mu T^L \rceil$, if it is smaller than $\lceil \mu T^L \rceil - \mu T^L$. Given that the service time is deterministic and equal to $1/\mu$, the distribution of the remaining completion time of the outstanding replenishment order in service at the beginning of the time interval is uniformly distributed over the interval $[0,1/\mu]$; therefore, the probability that the percentage of the remaining completion time of the outstanding replenishment order in service at the beginning of the time interval is greater than $\lceil \mu T^L \rceil - \mu T^L$ is equal to than $\lceil \mu T^L \rceil - \mu T^L$, whereas the probability that it is less than $\lceil \mu T^L \rceil - \mu T^L$ is $1 - (\lceil \mu T^L \rceil - \mu T^L) = \mu T^L - \lfloor \mu T^L \rfloor$. This means that

$$P_{T_k^L}(i) = \begin{cases} \lceil \mu T^L \rceil - \mu T^L, & i = \lfloor \mu T^L \rfloor, \\ \mu T^L - \lfloor \mu T^L \rfloor, & i = \lceil \mu T^L \rceil, \\ 0, & \text{otherwise.} \end{cases}$$

Substituting the above expression in (9) yields the following expression for $P_Z(n)$ in terms of $P_N(\cdot)$, for $n > 0$:

$$P_Z(n) = (\lceil \mu T^L \rceil - \mu T^L)P_N(n + \lfloor \mu T^L \rfloor) + (\mu T^L - \lfloor \mu T^L \rfloor)P_N(n + \lceil \mu T^L \rceil),\ n > 0.$$

$C(T;S,L)$ and $S^*(T;L)$ can then be computed numerically from expressions (4) and (6), respectively, after rewriting them in a convenient form that involves $P_Z(n)$, for $n > 0$, and substituting $P_Z(n)$ with the above expression, as follows:

$$C(T;S,L) = h\left[S\left(1 - \sum_{n=S}^{\infty} P_Z(n)\right) - E[Z] + \sum_{n=S+1}^{\infty} nP_Z(n)\right] + b\sum_{n=S+1}^{\infty}(n-S)P_Z(n),$$

$$1 - F_Z(S) = \sum\nolimits_{n=S+1}^{\infty} P_Z(n) \le h/(h+b),$$

where

$$E[Z] = E[N] - E[M] = \rho + \frac{\rho^2}{2(1-\rho)} - \lambda T^L.$$

Finally, $C(T;S,L)$ can be numerically optimized with respect to L after replacing S with $S^*(T;L)$ in the expression above. Given that the $M/D/1$ system satisfies Assumption 1, the optimal planned supply lead time L^* can be alternatively computed from (10), where $F_W(w)$ can be computed numerically as a limiting case of the $M/E_k/1$ system [12]. For the $M/D/1$ system it is well-known that $E[W] = 1/\mu + \rho^2/2(1-\rho)\lambda$. (e.g. see [12]).

Case 3: *M/M/∞*. In case the supply process is modeled as an infinite-server station, where each server has exponential service rate μ, by Palm's Theorem, N has a Poisson distribution with mean $\lambda E[W] = \lambda/\mu$, i.e. $P_N(n) = e^{-\rho}\rho^n/n!$, $n = 0, 1, \ldots$, where $\rho = \lambda/\mu$ [12]. If $T = 0$, the optimal base stock S^* can be computed numerically by (8), after substituting $P_N(n)$ with the above expression. Moreover, the following non closed-form expression for the steady-state distribution of X is developed in [4]:

$$P_X(n) = \begin{cases} \displaystyle\sum_{k=0}^{\infty} \frac{\alpha^{k+S-n}\beta^k}{(k+S-n)!k!} e^{-(\alpha+\beta)}, & n < S, \\ \displaystyle\sum_{k=0}^{\infty} \frac{\alpha^k \beta^{k+n-S}}{k!(k+n-S)!} e^{-(\alpha+\beta)}, & n \ge S, \end{cases}$$

where $\alpha = \rho e^{-\mu\min(T,L)}$, $\beta = \rho[\mu\min(T,L) - 1 + e^{-\mu\min(T,L)}]$ and $\rho = \lambda/\mu$. $C(T;S,L)$ and $S^*(T;L)$ can then be computed numerically from (3) and (5) after substituting $P_X(n)$ with the above expression. Finally, $C(T;S,L)$ can be numerically optimized with respect to L after replacing S with $S^*(T;L)$. Note that since the $M/M/\infty$ system does not satisfy Assumption 1, the optimal planned supply lead time L^* can not be computed from (10), even though $F_W(w)$ is known and is given by $F_W(w) = 1 - e^{-\mu w}$.

Case 4: *M/D/∞*. In case the supply process is modeled as a infinite-server station, where each server has deterministic service time $1/\mu$, by Palm's Theorem, N is Poisson distributed with mean $\lambda E[W] = \lambda/\mu$, i.e. $P_N(n) = e^{-\rho}\rho^n/n!$, $n = 0, 1, \ldots$, where $\rho = \lambda/\mu$. If $T = 0$, the optimal base stock S^* can be computed numerically by (8), after substituting $P_N(n)$ with the above expression. The result is of course identical to that for the $M/M/\infty$ system, given that the two systems have the same ρ. It is clear that any system with deterministic service time $1/\mu$ and demand lead time T, such that $T \le \mu$, is equivalent to a system with deterministic service time $1/\mu - T$ and zero demand lead time (see also [13]). $C(T;S,L)$ and $S^*(T;L)$ can then be computed numerically from (7) and (8), respectively. Clearly, when $T = 1/\mu$, the deterministic service time in the equivalent system is zero and therefore $S^*(T;L)$ and $C(T;S^*(T;L),L)$ are zero too. This implies that

$$L^* = 1/\mu. \tag{14}$$

Given that the $M/D/\infty$ system satisfies Assumption 1, L^* can be alternatively computed in closed form from (10), where $F_W(w) = 1$, if $w \ge 1/\mu$, and $F_W(w) = 0$, otherwise. The result is identical to that given by (14).

3 Single-Stage Make-to-Stock Supply System with Variable Unreliable ADI

We extend the basic model with constant, reliable ADI described in Section 2 to investigate the impact of variability and uncertainty in the amount of ADI. In the extended model, we assume that there are two classes of customers. Each customer in the first class requires immediate service (rush job). Each customer in the second class makes a reservation for a finished product a fixed amount of time, T, before his/her requested due date and must confirm (or cancel) this reservation a fixed amount of time, Δ, prior to his/her due date. T and Δ are referred to as the *demand lead time* and *confirmation lead time*, respectively, and are the same for all customers in the second class. A customer in the second class may cancel or confirm his/her reservation with stationary probability q and $1 - q$, respectively; q is referred to as the *cancellation percentage*. If a customer cancels his/her order, a stack called *cancelled reservations stack* (CRV) is increased by one. Finally, an arriving customer may belong to the first or second class with stationary probability p and $1 - p$, respectively; p is referred to as the *rush-job percentage*. Parameters p, q and T are *system* parameters characterizing the variability, uncertainty and extent of ADI. Parameters S, L and Δ, are *design* parameters of the replenishment control policy. They directly affect customer service and therefore indirectly affect customer behavior, i.e. λ, p and q. In fact, Δ also has a direct effect on customer behavior as well. Customer demands for end-items in FG inventory arrive according to Poisson process with effective arrival rate

$$\lambda_e = \lambda[p + (1 - p)(1 - q)]. \tag{15}$$

The arrival of a customer demand does not necessarily trigger the issuing of a replenishment order. The decision of whether to issue or not a replenishment order is taken $\max(0, T - \max(L, \Delta))$ time units after the arrival of a customer demand and is based on the current state of the CRV. Namely, if the CRV is empty, a replenishment order is issued. Otherwise, a replenishment order is not issued and the CRV is decreased by one. If it is decided to issue a replenishment order, the timing of issuing this order is still determined by offsetting the demand due date by L, according to an MRP time-phasing logic. A schematic representation of the system is shown in Figure 1, where big circles represent delays, small circles represent probabilistic routing, and squares represent decision points.

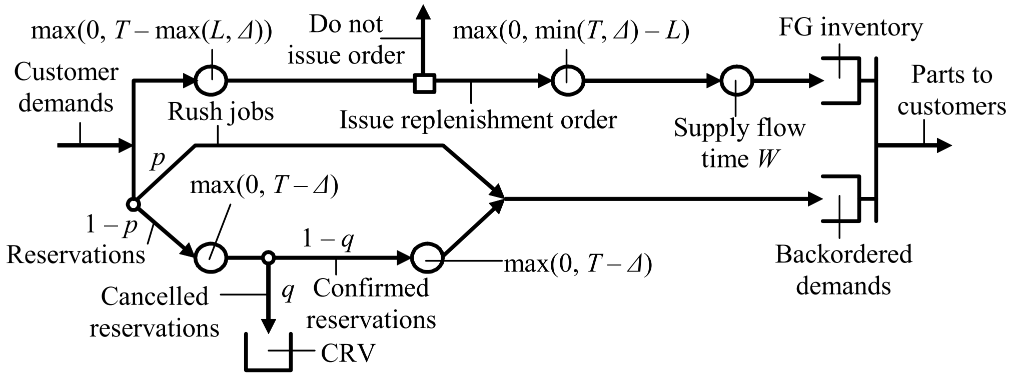

Figure 1: Single-stage make-to-stock supply system with variable unreliable ADI.

4 Numerical Results

In this section, we illustrate the analytical results developed in Section 2, and we investigate the impact of the system parameters p and q as well as the control policy design parameter Δ (independent parameters) on the optimal values of the other design parameters S and L (dependent parameters) for the system with variable unreliable ADI presented in Section 3. Tables 1-4 show the independent and resulting dependent parameters for several instances of the four different supply cases analyzed in Section 2. The cost rates h and b are the same in all instances and equal 1 and 9, respectively. The supply rate μ is set so that $E[W] = 5$ for all instances, and the demand rate λ is set so that $\lambda_e = 0.8$ for all instances, where λ and λ_e are related by (15). Using the analytical expressions presented in Section 2, we computed the optimal parameters L^* and $S^*(T) = S^*(T;L^*)$ for $T = 0$ and L^*, as well as the respective minimum long run expected average cost, $C^*(T) = C(T;S^*(T),L^*)$, for the instances where $p = 0$ and $q = 0$ (shaded rows of Tables 1-4), which correspond to the system with constant reliable ADI discussed in Section 2. For all other instances, we used optimization via simulation.

Instance	*p*	*q*	*Δ*	$S^*(0)$	$C^*(0)$	$S^*(L^*)$	L^*	$C^*(L^*)$
1	0	0	-	10	10.3188	0	11.5129	9.210
2		0.15	0				10	9.440
3			5				10	9.409
4			8				11	9.258
5		0.35	0				8	9.882
6			5				9	9.548
7			8				10	9.411
8	0.2	0	-				14	9.222
9		0.15	0				12	9.519
10			5				13	9.405
11			10				13	9.401
12		0.35	0				10	9.923
13			5				12	9.699
14			10				13	9.523
15	0.5	0	-				22	9.390
16		0.15	0				21	9.607
17			5				22	9.543
18			10				22	9.522
19			15				23	9.465
20			20				24	9.359
21		0.35	0				19	9.986
22			5				20	9.912
23			10				22	9.810
24			15				24	9.698
25			20				25	9.327
26	0.7	0	-				38	9.669
27		0.15	0				37	9.864
28			5				37	9.777
29			10				38	9.723
30			15				39	9.717
31			20				40	9.703
32		0.35	0				34	10.047
33			5				36	10.045
34			10				37	10.003
35			15				39	9.948
36			20				41	9.940

Table 1: Case 1 – Parameters for a single-stage make-to-stock *M*/*M*/1 supply system with constant, unreliable ADI, operating under a BSADI policy ($\mu = 1$, $\lambda_e = 0.8$, $h = 1$, $b = 9$).

Instance	*p*	*q*	*Δ*	$S^*(0)$	$C^*(0)$	$S^*(L^*)$	L^*	$C^*(L^*)$
37	0	0	-	9	8.9968	0	9.4164	7.693
38	0.2						12	7.857
39	0.5						20	7.866
40	0.7						34	8.066

Table 2: Case 2 – Parameters for a single-stage make-to-stock *M*/*D*/1 supply system with constant, unreliable ADI, operating under a BSADI policy ($\mu = 0.9124$, $\lambda_e = 0.8$, $h = 1$, $b = 9$).

Instance	*p*	*q*	*Δ*	$S^*(0)$	$C^*(0)$	$S^*(L^*)$	L^*	$C^*(L^*)$
41	0	0	-	7	3.8476	2	5	2.964
42	0.2					3	5	3.161
43	0.5					4	6	3.444
44	0.7					5	6	3.598

Table 3: Case 3 – Parameters for a single-stage make-to-stock *M*/*M*/∞ supply system with constant, unreliable ADI, operating under a BSADI policy ($\mu = 1/5$, $\lambda_e = 0.8$, $h = 1$, $b = 9$).

Instance	p	q	Δ	$S^*(0)$	$C^*(0)$	$S^*(L^*)$	L^*	$C^*(L^*)$
45	0.0	0.00	-	7	3.8476	0	5	0
46		0.15	0				5	0.881
47			3				5	0.453
48		0.35	0				5	2.680
49			3				5	1.351
50	0.2	0.00	-			2	5	1.799
51		0.15	0				5	2.262
52			3				5	2.070
53		0.35	0			1	5	2.905
54			3			2	5	2.539
55	0.5	0.00	-			4	5	2.750
56		0.15	0				5	3.046
57			3				5	2.951
58		0.35	0				5	3.521
59			3				5	3.247
60	0.7	0.00	-			5	5	3.207
61		0.15	0				5	3.396
62			3				5	3.352
63		0.35	0				5	3.671
64			3				5	3.543

Table 4: Case 4 – Parameters for a single-stage make-to-stock $M/D/\infty$ supply system with constant, unreliable ADI, operating under a BSADI policy ($\mu = 1/5$, $\lambda_e = 0.8$, $h = 1$, $b = 9$).

The results in Tables 1-4 lead to the following observations.

(a) For $T = 0$, $S^*(0)$ and $C^*(0)$ are lower in the uncapacitated cases 3 and 4 than in their respective capacitated cases 1 and 2. Moreover, within the capacitated cases 1 and 2, $S^*(0)$ and $C^*(0)$ are lower in the deterministic-supply-time case 2 than in its respective exponential-supply-time case 1. For the uncapacitated cases 3 and 4, $S^*(0)$ and $C^*(0)$ are the same.

(b) In all instances of cases 1 and 2 (instances 1-35 and 37-40) and in those instances of case 4 where $p = 0$ (instances 45-49), $S^*(L^*) = 0$. In all instances of case 3 (instances 41-44) and in those instances of case 4 where $p > 0$ (instances 50-64), $S^*(L^*) > 0$. The difference between the two groups of instances is that in the first group of instances (instances 1-35, 37-40, 45-49) customer demands are always satisfied in the order of arrival, whereas in the second group of instances (instances 41-44, 50-64) customer demand are not necessarily satisfied in the order of arrival. This leads to the following very interesting conjecture.

If customer demands are satisfied in the order of arrival, safety stock and safety time are totally interchangeable, otherwise, they are not, where by "total interchangeability" we mean that when the demand lead time T switches from zero to L^*, the optimal supply policy switches from a pure make-to-stock policy into a pure make-to-order policy.

(c) In all instances of case 4 and in that instance of case 3 where $p = 0$ and $q = 0$ (instance 41), $L^* = E[W] = 5$. In all other instances, $L^* > E[W] = 5$. Note that in instance 42 of case 3, L^* appears to be equal to 5 too. This is because in our simulations, we evaluated integer values of L, whereas in reality L is continuous. We strongly suspect that in instance 42, the true (continuous) value of L^* is actually greater than 5.

(d) L^* and $C^*(L^*)$ are lower in the uncapacitated cases 3 and 4 than in their respective capacitated cases 1 and 2. Moreover, within the capacitated cases 1 and 2, L^* and $C^*(L^*)$ are lower in the deterministic-supply-time case 2 than in its respective exponential-supply-time case 1. Similarly, within the capacitated cases 3 and 4, L^* and $C^*(L^*)$ are lower in the deterministic-supply-time case 4 than in its respective exponential-supply-time case 3, except when $p = q = 0$, where they are equal to each other (see observation (c)).

(e) In all cases, as p increases, L^* and $C^*(L^*)$ increase and $S^*(L^*)$ increases or remains the same. This can be explained by the fact that as the rush-job percentage p increases, the amount of ADI decreases, and consequently the system cost and L^* increase.

(f) As q increases, $C^*(L^*)$ increases and L^* decreases. This can be explained as follows. As q increases, more reservations are cancelled; therefore, the unreliability in the amount of ADI and consequently the system cost increase. At the same time, on the average, more replenishment orders triggered by eventually cancelled reservations are issued. This tends to increase FG inventory and consequently decrease $S^*(T)$. As $S^*(T)$ decreases, L^* also decreases.

(g) As Δ increases, $C^*(L^*)$ decreases and L^* increases. This is the opposite of observation (f) and can be explained as follows. As Δ increases, customers are forced to confirm of cancel their reservations earlier; therefore, the unreliability in the amount of ADI and consequently the system cost decrease. At the same time, on the average, less replenishment orders triggered by eventually cancelled reservations are issued. This tends to decrease FG inventory and consequently increase $S^*(T)$. As $S^*(T)$ increases, L^* also increases.

(h) In all instances, as T increases, $S^*(T)$ and $C^*(T)$ decrease in a linear fashion. This is not imprinted in the results in Tables 1-4, which due to space considerations show only values of $S^*(T)$ and $C^*(T)$ at the extreme values $T = 0$ and $T = L^*$, but it has been observed in our full set of simulations. Figure 2 illustrates this observation for the four instances where the ADI is constant and reliable (instances 1, 37, 41, 45). Similar figures can be drawn for all other instances.

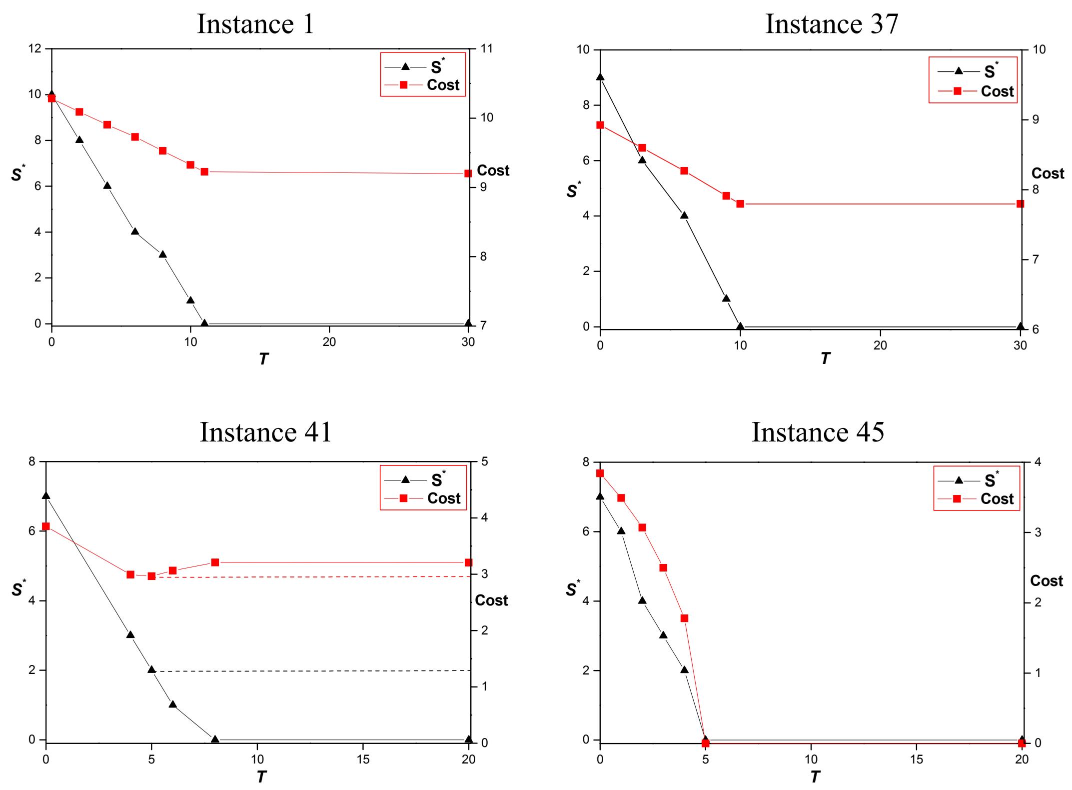

Figure 2: $S^*(T)$ and $C^*(T)$ vs. T for instances 1, 37, 41 and 45.

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