Inventory Control and Rationing in a System with Deterministic and Stochastic Sources of Demand *

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Abstract

We consider a periodic review inventory system with two priority demand classes, one deterministic and the other stochastic. The deterministic demand must be met immediately in each period. However, the units of stochastic demand that are not satisfied during the period when demand occurs are treated as lost sales. At each decision epoch, one has to decide not only whether an order should be placed and how much to order but also how much demand to fill from the stochastic source. The firm has the option to ration inventory to the stochastic source (i.e., not satisfy all customer demand even though there is inventory in the system). We characterize the structure of the optimal policy and show that in general the optimal order quantity and rationing policy are state dependent and do not have a simple structure.

1 Introduction and Motivation

We consider a firm which supplies goods to two different types of customers: (1) customers who have long term supply contracts and (2) customers who request goods occasionally. The orders of the customers who have supply contracts are known in advance and can be appropriately modeled as deterministic. On the other hand, unexpected requests from occasional customers are unknown until they are received and are most appropriately modeled as stochastic. Due to the stochastic nature of the demand, unsatisfied orders are unavoidable. To some extent, most real-world distribution systems operate in this fashion although the degree of randomness varies.

We have observed many cases of such distribution or manufacturing systems with multiple sources of demand in practice. For example, a large glass manufacturer in the Detroit area signs contracts with the Big Three Auto Companies to provide them with glass for their current year models. These long-term contracts require just-in-time deliveries and the manufacturer faces significant penalties if it misses a delivery. The manufacturer also faces demand from the after-market segment (i.e., automotive glass replacement for installation into older vehicles). The manufacturer has the option to accept or reject the after-market orders (see Carr and Duenyas (1997) for further details). We have also observed supply contracts that specify a minimum guaranteed order quantity every period. In these contracts, the buyer guarantees to purchase a specified amount each period but has the

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flexibility to order larger amounts. However, the supplier is only required to deliver the specified amount and is expected to make an effort to supply the rest.

There has been an explosion of recent work on supply contracts. Tsay et al. (1998) provide a review of supply chain contracts. A recent review of supply contracts with quantity commitments is provided by Anupindi and Bassok (1998). Using their terminology, in the problem we address, we assume that the firm has secured a given amount of stationary commitments for each period. The supplier has to provide 100% service level on the committed orders but is allowed to lose sales on the additional stochastic demand that he faces. Thus, the firm faces two priority demand classes. Demand from class 1 customers is deterministic and stationary and has to be completely satisfied every period. Demand from class 2 customers is stochastic and the firm is allowed to lose sales on those orders.

There are two simplistic extremes for managing such a system in practice. At one extreme, one can aggregate the two sources of stochastic and deterministic demand and treat the combined total as if demand arrives from only one source. However, adjustments need to be made to make sure that the deterministic demand is always satisfied. (i.e., all demand is satisfied in every period but the firm makes sure to order whenever its inventory is in danger of falling below the amount required to satisfy the deterministic demand). Another extreme is to completely separate management of the deterministic and stochastic sources of demand. The firm uses an EOQ-type policy for the deterministic demand and keeps a separate inventory for the stochastic demand which is governed by an (s,S) type policy. While this approach leads to feasible practice, it incurs significant extra ordering costs and is clearly suboptimal. The optimal policy lies between these two extreme policies. For any given state, an optimal policy would specify if an order should be placed, its quantity and how much of the stochastic demand to fill (i.e., the optimal rationing policy).

There has been some work on supply contracts with stationary commitments. We again refer the reader to Anupindi and Bassok (1998) for a review but would like to mention two papers particularly related to our model. Anupindi (1994) analyzes a situation where a buyer commits to buying a given amount of inventory from the supplier in the beginning of the planning horizon but has the option to dynamically adjust quantities. However, his focus is on the buyer who has to solve the problem of what quantity to commit to and how much to adjust in each period (given that committed quantities cost less and adjustments are more expensive). Our focus is on the supplier who has to supply a contract with a stationary commitment. Moinzadeh and Nahmias (1999) consider the same problem where only an upward adjustment is allowed. Once again, the focus is on developing approximations for the optimal commitment quantity.

There has been some work on inventory systems with priority demand classes. Topkis(1968) considers an inventory model with several demand classes and shows that under certain conditions the optimal ordering policy can be characterized as a base stock policy and the optimal rationing policy can be specified by a set of control limits. However, his model does not include setup costs. Nahmias and Demmy (1981) model an inventory system with both high and low priority demands and backorders. They analyze a control policy that backorders demands from low priority customers when the inventory level reaches a critical threshold and compute fill rates when there is rationing and no rationing. Their suggested policy bears some similarities to our heuristic (s,k,S) policy. However, their problem is different since 1) they do not have a class of customers all of whose demands have to be satisfied, and 2) their focus is on performance evaluation rather than characterizing the structure of the optimal policy. Cohen et al. (1988) model an (s, S) inventory system with two priority demand classes. Assuming that demand that can not be satisfied from stock is lost and no stock is reserved to meet future demand of high priority customers, they develop a heuristic for computing an (s, S) policy that minimizes expected costs subject to a service-level constraint. Sobel and Zhang (1997) model a system with non-stationary stochastic and deterministic demand similar to ours assuming that stochastic demand is always met if there is stock available (i.e., no rationing is allowed) and show that a modified (s, S) policy is optimal under general conditions. The fact that we allow the firm to ration inventory to the lower priority customers is the factor that significantly complicates and differentiates our paper from those of Sobel and Zhang and Cohen et al. In two recent papers, Ha (1997a, 1997b) modeled an M/M/1 make-to-stock queue with multiple customer classes where the firm is allowed to ration inventory. However, his models do not have any fixed order/production costs, and the M/M/1 assumptions imply that demand by all classes of distributions arrive according to exponential inter-arrival distributions.

In this paper, we provide a dynamic programming formulation of the problem. We are able to characterize the structure of the optimal joint policy (ordering and rationing policies), but we demonstrate that it is state dependent and does not have a simple structure. In Frank, Zhang and Duenyas (1999), this complexity leads us to develop a heuristic policy that shares many of the characteristics of the optimal policy. This policy is specified by three numbers, (s, k, S), with (s, S) specifying the order policy and k specifying the rationing policy. In that paper, we test this policy thoroughly and show that it performs very well.

The remainder of this document is organized as follows. In Section 2, we introduce the dynamic programming formulation of the problem. In Section 3, we characterize the structure of the optimal policy. Section 4 concludes the paper.

2 Problem Formulation

We consider a periodic review system with two types of demand, deterministic and stochastic. In this system, the deterministic demand must be satisfied immediately while the units of stochastic demand that are not met in the same period are lost. Demand arrives at the beginning of each period and an order (if any) is then placed. The quantity ordered arrives by the end of the period when the firm satisfies the deterministic demand for that period as well as the portion of stochastic demand it chooses to satisfy. The objective is to determine the quantity to order and the amount of stochastic demand to satisfy for any given initial inventory level and current stochastic demand so as to minimize the expected total discounted cost.

We first introduce the following notation:

K	setup cost per order
h	holding cost per unit per period
c	variable purchasing cost per unit
P	selling price per unit of stochastic demand ($P \geq c$)
π	lost sales cost per unit per period
λ_d	deterministic demand (stationary commitment) in each period
p_i	probability that the stochastic demand in each period is i
β	discount factor ($\beta \leq 1$)

x	inventory on hand at the beginning of a period
y	current period stochastic demand
Q	number of units to order
w	number of units of stochastic demand to satisfy

The objective is to determine a policy (Q, w) that achieves the minimum expected total discounted cost for any given state (x, y). Let $V_n(x, y)$ be the optimal expected total discounted cost given that the current state is (x, y) and there are n periods to go. We can then formulate the finite horizon problem using the following dynamic program.

$$\begin{aligned} V_n(x,y) = \min_{Q,w} \quad \{ \quad & K\sigma(Q)^+ + cQ + h(x+Q-\lambda_d-w) + \pi(y-w) - Pw \\ & +\beta \sum_i p_i V_{n-1}(x+Q-\lambda_d-w, i); \\ & Q \geq 0; \;\; 0 \leq w \leq \min\{y, x+Q-\lambda_d\} \;\; \} \end{aligned} \tag{2.1}$$

where $\sigma(Q)^+ = 1$ if $Q > 0$, and 0 otherwise; and $V_0(\cdot, \cdot) = 0$.

We note that in our formulation, we ignore the revenue gained by satisfying the contracted deterministic demand in each period since these orders have to be completely satisfied by any feasible policy. Moreover, our model allows different selling price for committed units. In (2.1), setup costs are incurred every time an order is placed and penalty costs are incurred for the portion of stochastic demand that is not satisfied.

As $n \to \infty$, $V_n \to V$, and by replacing $V_n(x, y)$ and $V_{n-1}(x, y)$ by $V(x, y)$, we obtain the dynamic programming equations for the infinite-horizon discounted cost case:

$$\begin{aligned} V(x,y) = \min_{Q,w} \quad \{ \quad & K\sigma(Q)^+ + cQ + h(x+Q-\lambda_d-w) + \pi(y-w) - Pw \\ & +\beta \sum_i p_i V(x+Q-\lambda_d-w, i); \\ & Q \geq 0; \;\; 0 \leq w \leq \min\{y, x+Q-\lambda_d\} \;\; \} \end{aligned} \tag{2.2}$$

We note that if $\lambda_d = 0$, (i.e., there is only stochastic demand with lost sales allowed), formulation (2.1) can be replaced by a dynamic program with one state variable $x - y$ and one decision variable Q and the optimal policy is an (s, S) policy under general conditions. The presence of the deterministic demand $\lambda_d > 0$ which has to be satisfied every period significantly complicates the problem and the structure of the optimal solution. We next characterize the structure of this optimal policy.

3 Characterization of the Structure of the Optimal Policy

We first present a series of theorems which thoroughly characterizes the structure of the optimal policy. We then provide some numerical examples that help the reader visualize this optimal structure.

A complete policy is characterized by the following two decision variables: the order quantity, Q, and the amount of stochastic demand to fill, w. These two decision variables are dependent on the state variables x and y which represent the inventory on hand and the amount of stochastic demand for the current period, respectively. (It is clear that Q and w are functions of the state (x, y) as well as the period n. However, instead of writing $Q_{x,y,n}$, we will in the following suppress the dependence of Q on one or more of the variables depending on the context. For example, when comparing order quantities in the same period but different states, we will write $Q_{x,y}$. The reader should keep in mind the dependence of Q and w on all three variables at all times).

We will prove the optimal policy for the infinite horizon case by obtaining it as the limit of the policy for the finite horizon case. That is, we will first use induction to prove the structure of the optimal policy for the finite horizon problem and then argue that the same structure holds for the infinite horizon problem. (We have omitted all the proofs. See Frank, Zhang and Duenyas (1999) for the proofs).

Define $f_n(x, y, Q, w)$ as the optimal expected discounted cost when the initial state is (x, y) and a feasible policy (Q, w) is used with n periods to go (in the following, for brevity, we will write period n instead of n periods to go) i.e.,

$$\begin{aligned} f_n(x, y, Q, w) &= K\sigma(Q)^+ + cQ + h(x + Q - \lambda_d - w) + \pi(y - w) - Pw \\ &\quad + \beta \sum_i p_i V_{n-1}(x + Q - \lambda_d - w, i) \end{aligned}$$

where $Q \geq 0$ and $0 \leq w \leq \min\{y, x + Q - \lambda_d\}$.

Theorem 1 *In any state (x, y), if $Q > 0$, then $x + Q - \lambda_d \geq y$ and $w = y$. That is, if we order in a period, we always order enough to satisfy all stochastic demand as well as the deterministic demand.*

Theorem 2 *If $x \geq \lambda_d + y$, then it is optimal not to order. That is, if current inventory is sufficient to satisfy both the deterministic and stochastic demand, then no order should be placed.*

The next two theorems provide descriptions of the relationships between the order quantities and rationing policies (i.e., how much of stochastic demand to satisfy) in different states.

Theorem 3 *If it is optimal to order $Q_y > 0$ in state (x, y), then it is optimal to order in state $(x, y + 1)$, and $Q_{y+1} = Q_y + 1$.*

Corollary 1 *There exists a threshold $\hat{y}_x$ for any x such that it is optimal to order if $y \geq \hat{y}_x$ ($\hat{y}_x = 0$ if $x < \lambda_d$, $\hat{y}_{\lambda_d} \geq 1$, and $\hat{y}_x \geq x - \lambda_d$ for $x > \lambda_d$) and the order quantities increase by one unit as y increases. It is optimal not to order when $y < \hat{y}_x$. Define $s_x = x - \hat{y}_x - \lambda_d + 1$. Then for each x, there exists a level s_x such that it is optimal to order if $x - y - \lambda_d < s_x$.*

Whereas Corollary 1 just states the existence of a threshold for ordering to be optimal for each inventory level x, the next two theorems prove that in states where it is optimal to order, one always orders enough to bring inventory up to the same order-up-to level. (Note that the order-up-to levels can be different in different periods in a finite horizon problem).

Theorem 4 *In a given period, if policy $(Q_{x,y}, w_{x,y})$, $w_{x,y} > 0$, is optimal in state (x, y), $x > 0, y > 0$, then policy $(Q_{x-1,y-1}, w_{x-1,y-1})$, where $Q_{x-1,y-1} = Q_{x,y}$ and $w_{x-1,y-1} = w_{x,y} - 1$, is optimal in state $(x - 1, y - 1)$.*

Corollary 2 *In a given period, if it is optimal to order $Q > 0$ in state $(x+1, y+1)$, then it is optimal to order the same amount Q in state (x, y).*

Corollary 2 also implies that s_x is nonincreasing in x. (Below, in Theorem 7, we derive a sufficient condition under which $s_x = s$ for all x). Combining Theorems 3, 4 and Corollary 2, we have the following result:

Theorem 5 *In any period n and state (x, y), if an order is placed, then the amount ordered will always bring the inventory to an optimal order-up-to level, $S_n = x + Q_{x,y} - \lambda_d - y$.*

The previous results have demonstrated that there exists a threshold $\hat{y}_x$ such that it is optimal to order whenever $y \geq \hat{y}_x$ and that whenever it is optimal to order, it is optimal to order up to S_n. However, the results do not tell us how the optimal policy behaves when it is not optimal to order. The next theorem characterizes the optimal rationing policy when it is optimal not to order in a given period. (Recall that we showed above that satisfying all stochastic demand is optimal when one orders). The result shows that when it is optimal not to order, the rationing policy either satisfies all the stochastic demand or results in remaining inventory at the end of the period to be an integer multiple of deterministic demand. This result significantly cuts down on the number of possibilities one needs to consider for the rationing policy.

Theorem 6 *In any given period n, if it is optimal not to order in state (x, y), then the optimal policy is $(0, w)$ where $w \in \{y, x - \lambda_d - k_{x,y}\lambda_d\}$ and $k_{x,y}$ is a nonnegative integer.*

The previous results have shown the existence of an order-up-to level when it is optimal to order. Further, it is always optimal to order when $x < \lambda_d$ due to the fact that the deterministic demand has to be satisfied. Also, we have shown that for each x, there exists a reorder point s_x. The next theorem characterizes a sufficient condition under which $s_x = s$, i.e., the reorder point is independent of x.

Theorem 7 *If it is optimal to order in state (x, y), $x \geq \lambda_d$, then it is also optimal to order in state $(x+1, y+1)$ if $(\pi + h + P - \beta c)\lambda_d > \beta K$.*

Using Theorems 1 through 7, and the convergence of the finite horizon discounted cost functions to the infinite horizon discounted cost function as the number of periods goes to ∞, we are now ready to characterize the structure of the optimal policy for the finite and infinite horizon discounted cases.

Theorem 8 *For the finite horizon discounted problem*

1. *If $(\pi + h + P - \beta c)\lambda_d > \beta K$, then there exist values (s_n, S_n) such that it is optimal to order if $x < \lambda_d$, or $x - y - \lambda_d \geq s_n$. Furthermore, when it is optimal to order, one always orders $S_n - x + y + \lambda_d$ units. If $(\pi + h + P - \beta c)\lambda_d \leq \beta K$, then the order-up-to policy remains the same but there exist values $s_{n,x}$ such that $s_{n,x}$ depend on the inventory on hand, x (from Theorem 7 and Corollary 1).*

2. *In periods when it is optimal to order, all stochastic demand is satisfied. However, when it is not optimal to order, there exist nonnegative integer values* $k_{n,x,y}$ *for each state* (x, y) *in period* n *such that the optimal amount of stochastic demand to satisfy is given by* $w_n(x, y) = \min\{y, x - (k_{n,x,y} + 1)\lambda_d)\}$.

For the infinite horizon discounted problem, the same results hold with $(s_{n,x}, k_{n,x,y}, S_n)$ *replaced by* $(s^*_x, k^*_{x,y}, S^*)$.

Having characterized the structure of the optimal policy in Theorem 8, we demonstrate the kinds of behavior that Theorem 8 predicts in Tables 1-3. In the three examples demonstrated in Tables 1-3, we obtained infinite horizon optimal policies using the MDP formulation of the problem. The example in Table 1 shows typical behavior predicted by our analytical results. In this case, demand is discrete uniform between 0 and 10 (shown by U[0,10] in Table 1). The optimal policy is characterized by $(s^*, S^* = -5, 12)$. That is, the policy is always to order whenever $x < \lambda_d = 5$ or $x - y - \lambda_d < s^* = -5$. Furthermore, whenever it is optimal to order, one orders a quantity $Q^*_{x,y}$ to raise the inventory to level $S^* = 12 = Q^*_{x,y} + x - y - \lambda_d$. Note that whenever it is optimal to order, one always satisfies all the stochastic demand. However, when it is not optimal to order, it is optimal to satisfy either all the stochastic demand that it is possible to satisfy or a quantity that leaves a multiple of λ_d in inventory. Consider the case where $x = 15$. In this case, if stochastic demand y, is between 0 and 5, it is optimal to satisfy all of it. When $y = 6$, the policy decides to satisfy only 5 units of stochastic demand, thus reserving 5 units for next period's deterministic demand. However, for $y > 6$, it is again optimal to satisfy all stochastic demand. In this case, the penalty of not satisfying the stochastic demand has increased so that it is higher than the possible savings that one would obtain by reserving inventory to postpone ordering by one period.

Whereas in the example in Table 1, it is optimal to reserve at most one period's worth of deterministic demand, in the example in Table 2, it is optimal to reserve multiple periods' worth. In this example, stochastic demand is discrete uniform between 5 and 15. In this case, the policy is again characterized by (s^*, S^*), but $s^* = -\infty$ and $S^* = 17$. Note that in this case, $s^* = -\infty$ implies that, we only order when $x < \lambda_d = 2$. When $x = 6$, the optimal policy is to reserve all inventory for deterministic demand regardless of the level of stochastic demand. Note that since $\lambda_d = 2$, this implies that when $x = 6$, the policy decides to save enough to satisfy the deterministic demand for the following two periods to postpone ordering. This is not surprising since K is so high and $P - c$ and π, which represent the lost opportunity cost and penalty for not satisfying stochastic demand respectively, are relatively small.

It is interesting to note that although the inequality $(\pi + h + P - \beta c)\lambda_d > \beta K$ is not satisfied in the example in Table 1, it was still optimal to order whenever $x - y - \lambda_d < s^*$. This is because the result is a sufficient (but not a necessary) condition for s^* to be independent of x. In most of the examples we have looked at, we have found that an s^* independent of x exists such that it is optimal to order whenever $x < \lambda_d$ or $x + y - \lambda_d < s^*$ even when this inequality is not satisfied. However, the example in Table 3 shows that as Theorem 8 predicts, there are cases when s^*_x will be dependent on x. (In this example, y varies between 0 and 14, but we only display the results for y between 5 and 14). Here, $s^*_2 = s^*_3 = -10$ and $s^*_4 = -11$.

The previous results characterize the policy to follow when it is optimal to order and further tell us when one needs to order (when $(\pi + h + P - \beta c)\lambda_d > \beta K$). However, they do not tell us when it is optimal to reserve units for future deterministic demand and how much to reserve. Our characterization of the optimal rationing policy tells us that when no order is placed, the optimal amount of stochastic demand to satisfy is either all the demand or $x-(k_{x,y}+1)\lambda_d$ for some nonnegative integer $k_{x,y}$. Although this characterization gives us some insight into the structure,we would still need to solve the MDP to obtain the optimal values of $k_{x,y}$ for each state (x, y). Furthermore, it is not easy to describe and implement such a policy in practice. For this reason, using the intuition gained from the structure we observed, we introduce and test a much simpler policy in Frank, Zhang and Duenyas (1999).

Table 1: Example of optimal policies (Q, w)

Parameters $K = 25$, $\beta = 0.95$, $h = 1$,$\pi = 2$, $c = 4$, $P = 6$, $\lambda_d = 5$, $U[0, 10]$

$x\backslash y$	0	1	2	3	4	5	6	7	8	9	10
0	(17, 0)	(18, 1)	(19, 2)	(20, 3)	(21, 4)	(22, 5)	(23, 6)	(24, 7)	(25, 8)	(26, 9)	(27,10)
1	(16, 0)	(17, 1)	(18, 2)	(19, 3)	(20, 4)	(21, 5)	(22, 6)	(23, 7)	(24, 8)	(25, 9)	(26,10)
2	(15, 0)	(16, 1)	(17, 2)	(18, 3)	(19, 4)	(20, 5)	(21, 6)	(22, 7)	(23, 8)	(24, 9)	(25,10)
3	(14, 0)	(15, 1)	(16, 2)	(17, 3)	(18, 4)	(19, 5)	(20, 6)	(21, 7)	(22, 8)	(23, 9)	(24,10)
4	(13, 0)	(14, 1)	(15, 2)	(16, 3)	(17, 4)	(18, 5)	(19, 6)	(20, 7)	(21, 8)	(22, 9)	(23,10)
5	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(18, 6)	(19, 7)	(20, 8)	(21, 9)	(22,10)
6	(0, 0)	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(18, 7)	(19, 8)	(20, 9)	(21,10)
7	(0, 0)	(0, 1)	(0, 2)	(0, 2)	(0, 2)	(0, 2)	(0, 2)	(0, 2)	(18, 8)	(19, 9)	(20,10)
8	(0, 0)	(0, 1)	(0, 2)	(0, 3)	(0, 3)	(0, 3)	(0, 3)	(0, 3)	(0, 3)	(18, 9)	(19,10)
9	(0, 0)	(0, 1)	(0, 2)	(0, 3)	(0, 4)	(0, 4)	(0, 4)	(0, 4)	(0, 4)	(0, 4)	(18,10)
10	(0, 0)	(0, 0)	(0, 2)	(0, 3)	(0, 4)	(0, 5)	(0, 5)	(0, 5)	(0, 5)	(0, 5)	(0, 5)
11	(0, 0)	(0, 1)	(0, 1)	(0, 3)	(0, 4)	(0, 5)	(0, 6)	(0, 6)	(0, 6)	(0, 6)	(0, 6)
12	(0, 0)	(0, 1)	(0, 2)	(0, 2)	(0, 4)	(0, 5)	(0, 6)	(0, 7)	(0, 7)	(0, 7)	(0, 7)
13	(0, 0)	(0, 1)	(0, 2)	(0, 3)	(0, 3)	(0, 5)	(0, 6)	(0, 7)	(0, 8)	(0, 8)	(0, 8)
14	(0, 0)	(0, 1)	(0, 2)	(0, 3)	(0, 4)	(0, 4)	(0, 6)	(0, 7)	(0, 8)	(0, 9)	(0, 9)
15	(0, 0)	(0, 1)	(0, 2)	(0, 3)	(0, 4)	(0, 5)	(0, 5)	(0, 7)	(0, 8)	(0, 9)	(0,10)
16	(0, 0)	(0, 1)	(0, 2)	(0, 3)	(0, 4)	(0, 5)	(0, 6)	(0, 6)	(0, 8)	(0, 9)	(0,10)
17	(0, 0)	(0, 1)	(0, 2)	(0, 3)	(0, 4)	(0, 5)	(0, 6)	(0, 7)	(0, 7)	(0, 9)	(0,10)
18	(0, 0)	(0, 1)	(0, 2)	(0, 3)	(0, 4)	(0, 5)	(0, 6)	(0, 7)	(0, 8)	(0, 8)	(0,10)
19	(0, 0)	(0, 1)	(0, 2)	(0, 3)	(0, 4)	(0, 5)	(0, 6)	(0, 7)	(0, 8)	(0, 9)	(0, 9)
20	(0, 0)	(0, 1)	(0, 2)	(0, 3)	(0, 4)	(0, 5)	(0, 6)	(0, 7)	(0, 8)	(0, 9)	(0,10)

4 Conclusions and Further Research

In this paper, we considered a periodic review inventory system with two priority demand classes, one stochastic, and the other deterministic. This model is particularly applicable to situations where the firm has stationary commitments in each period due to contracts it has signed. The firm can also satisfy demand that is random, but is not obligated to. We showed that the optimal policy has an interesting but complicated structure and characterized this structure analytically. We use our understanding of the structure of the policy to develop an effective heuristic in Frank, Zhang and Duenyas (1999).

Further research should address more complicated situations. We have explored the case with backorders instead of lost sales and found that our heuristic also performs well in that case although we were not able to prove the same structural results. It would be interesting to characterize the structure of the optimal policy in the case with backorders. Furthermore, our model treats the case

Table 2: Example of optimal policies (Q, w)

Parameters $K = 50$, $\beta = 0.95$, $h = 1, \pi = 1$, $c = 3$, $P = 4$, $\lambda_d = 2$, $U[5, 15]$

$x\backslash y$	5	6	7	8	9	10	11	12	13	14	15
0	(24, 5)	(25, 6)	(26, 7)	(27, 8)	(28, 9)	(29,10)	(30,11)	(31,12)	(32,13)	(33,14)	(34,15)
1	(23, 5)	(24, 6)	(25, 7)	(26, 8)	(27, 9)	(28,10)	(29,11)	(30,12)	(31,13)	(32,14)	(33,15)
2	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)
3	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)
4	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)
5	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)
6	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)
7	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)
8	(0, 2)	(0, 2)	(0, 2)	(0, 2)	(0, 2)	(0, 2)	(0, 2)	(0, 2)	(0, 2)	(0, 2)	(0, 2)
9	(0, 3)	(0, 3)	(0, 3)	(0, 3)	(0, 3)	(0, 3)	(0, 3)	(0, 3)	(0, 3)	(0, 3)	(0, 3)
10	(0, 4)	(0, 4)	(0, 4)	(0, 4)	(0, 4)	(0, 4)	(0, 4)	(0, 4)	(0, 4)	(0, 4)	(0, 4)
11	(0, 5)	(0, 5)	(0, 5)	(0, 5)	(0, 5)	(0, 5)	(0, 5)	(0, 5)	(0, 5)	(0, 5)	(0, 5)
12	(0, 5)	(0, 6)	(0, 6)	(0, 6)	(0, 6)	(0, 6)	(0, 6)	(0, 6)	(0, 6)	(0, 6)	(0, 6)
13	(0, 5)	(0, 6)	(0, 7)	(0, 7)	(0, 7)	(0, 7)	(0, 7)	(0, 7)	(0, 7)	(0, 7)	(0, 7)
14	(0, 5)	(0, 6)	(0, 7)	(0, 8)	(0, 8)	(0, 8)	(0, 8)	(0, 8)	(0, 8)	(0, 8)	(0, 8)
15	(0, 5)	(0, 6)	(0, 7)	(0, 8)	(0, 9)	(0, 9)	(0, 9)	(0, 9)	(0, 9)	(0, 9)	(0, 9)
16	(0, 5)	(0, 6)	(0, 7)	(0, 8)	(0, 9)	(0,10)	(0,10)	(0,10)	(0,10)	(0,10)	(0,10)
17	(0, 5)	(0, 6)	(0, 7)	(0, 8)	(0, 9)	(0,10)	(0,11)	(0,11)	(0,11)	(0,11)	(0,11)
18	(0, 5)	(0, 6)	(0, 7)	(0, 8)	(0, 9)	(0,10)	(0,11)	(0,12)	(0,12)	(0,12)	(0,12)
19	(0, 5)	(0, 6)	(0, 7)	(0, 8)	(0, 9)	(0,10)	(0,11)	(0,12)	(0,13)	(0,13)	(0,13)
20	(0, 5)	(0, 6)	(0, 7)	(0, 8)	(0, 9)	(0,10)	(0,11)	(0,12)	(0,13)	(0,14)	(0,14)
21	(0, 5)	(0, 6)	(0, 7)	(0, 8)	(0, 9)	(0,10)	(0,11)	(0,12)	(0,13)	(0,14)	(0,15)

where the deterministic demand in each period is the same. The case where the firm has commitments for a fixed number of future periods, but where these commitments can differ in value from period to period is an interesting but seemingly very difficult extension of this work.

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Table 3: Example of optimal policies (Q, w)

Parameters $K = 50$, $\beta = 0.9$, $h = 0.5$,$\pi = 0.5$, $c = 1$, $P = 3$, $\lambda_d = 2$, $U[0, 14]$

$x \backslash y$	5	6	7	8	9	10	11	12	13	14
0	(31, 5)	(32, 6)	(32, 7)	(32, 8)	(32, 9)	(32,10)	(32,11)	(32,12)	(32,13)	(32,14)
1	(30, 5)	(31, 6)	(31, 7)	(31, 8)	(31, 9)	(31,10)	(31,11)	(31,12)	(31,13)	(31,14)
2	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(30,11)	(30,12)	(30,13)	(30,14)
3	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(29,12)	(29,13)	(29,14)
4	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(28,14)
5	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)
6	(0, 2)	(0, 2)	(0, 2)	(0, 2)	(0, 2)	(0, 2)	(0, 2)	(0, 2)	(0, 2)	(0, 2)
7	(0, 3)	(0, 3)	(0, 3)	(0, 3)	(0, 3)	(0, 3)	(0, 3)	(0, 3)	(0, 3)	(0, 3)
8	(0, 4)	(0, 4)	(0, 4)	(0, 4)	(0, 4)	(0, 4)	(0, 4)	(0, 4)	(0, 4)	(0, 4)
9	(0, 5)	(0, 5)	(0, 5)	(0, 5)	(0, 5)	(0, 5)	(0, 5)	(0, 5)	(0, 5)	(0, 5)
10	(0, 5)	(0, 6)	(0, 6)	(0, 6)	(0, 6)	(0, 6)	(0, 6)	(0, 6)	(0, 6)	(0, 6)
11	(0, 5)	(0, 6)	(0, 7)	(0, 7)	(0, 7)	(0, 7)	(0, 7)	(0, 7)	(0, 7)	(0, 7)
12	(0, 5)	(0, 6)	(0, 7)	(0, 8)	(0, 8)	(0, 8)	(0, 8)	(0, 8)	(0, 8)	(0, 8)
13	(0, 5)	(0, 6)	(0, 7)	(0, 8)	(0, 9)	(0, 9)	(0, 9)	(0, 9)	(0, 9)	(0, 9)
14	(0, 5)	(0, 6)	(0, 7)	(0, 8)	(0, 9)	(0,10)	(0,10)	(0,10)	(0,10)	(0,10)
15	(0, 5)	(0, 6)	(0, 7)	(0, 8)	(0, 9)	(0,10)	(0,11)	(0,11)	(0,11)	(0,11)
16	(0, 5)	(0, 6)	(0, 7)	(0, 8)	(0, 9)	(0,10)	(0,11)	(0,12)	(0,12)	(0,12)
17	(0, 5)	(0, 6)	(0, 7)	(0, 8)	(0, 9)	(0,10)	(0,11)	(0,12)	(0,13)	(0,13)
18	(0, 5)	(0, 6)	(0, 7)	(0, 8)	(0, 9)	(0,10)	(0,11)	(0,12)	(0,13)	(0,14)
19	(0, 5)	(0, 6)	(0, 7)	(0, 8)	(0, 9)	(0,10)	(0,11)	(0,12)	(0,13)	(0,14)
20	(0, 5)	(0, 6)	(0, 7)	(0, 8)	(0, 9)	(0,10)	(0,11)	(0,12)	(0,13)	(0,14)

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